Agricultural Trade and Industrial Development

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February 11, 2026

Abstract

Is agricultural productivity conducive to economic development? We develop a two-country open-economy Schumpeterian growth model with endogenous takeoff. With both agricultural trade and a subsistence requirement, higher domestic agricultural productivity has ambiguous effects on the economy's takeoff and its transitional growth rate if domestic and imported agricultural goods are substitutes. Without the subsistence requirement, higher domestic agricultural productivity delays industrialization and lowers transitional growth by increasing domestic demand for agricultural labor. This specialization force works in the opposite direction of the change in domestic consumption pattern governed by the subsistence requirement, which tends to release labor from agriculture. Without agricultural trade, the specialization force is absent and the subsistence requirement on agricultural consumption implies that higher domestic agricultural productivity reallocates labor from agriculture to industry, hastening industrialization and raising transitional growth. Using cross-country panel data, we find that agricultural productivity has a positive effect on economic growth but this positive effect weakens and even becomes negative when reliance on agricultural imports is sufficiently high. Simulating the calibrated model, we find that improvement in domestic agricultural productivity accounts for about 30 percent and 15 percent of the changes in TFP growth in China and Japan, respectively, and more so for their main trading partner, the US.

JEL classification: O30, O40, F43
Keywords: international trade, agricultural productivity, innovation, endogenous takeoff

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1 Introduction

Is high agricultural productivity conducive to the industrial development of an economy? Early studies by Nurkse (1953) and Schultz (1953) argue that an improvement in agricultural productivity hastens the process of industrialization because it reallocates labor from farm to factory by changing the consumption pattern of households.[1] Subsequent studies by Mokyr (1976), Field (1978) and Wright (1979) argue that high agricultural productivity causes the economy to specialize in agricultural production and thus delays industrialization because it reallocates labor from factory to farm. Both of these theoretical predictions have received empirical support; see for example, Foster and Rosenzweig (2004, 2008) and Bustos *et al.* (2016, 2020). Thus far, the literature has found contrasting answers to our starting question because of the tension between two forces: reallocation from farm to factory due to the changing domestic consumption pattern and reallocation from factory to farm due to the changing specialization in international trade.

To make progress, this study develops an open-economy Schumpeterian growth model with endogenous takeoff that allows us to explore the different effects of agricultural productivity on the entire transition, from pre-industrial stagnation to modern innovation-driven growth, of economies engaged in international trade. In particular, the model has two countries, Home and Foreign, that trade both industrial and agricultural goods. Trade is asymmetric, however, in that Foreign does not import the Home agricultural good. Moreover, Home has a subsistence requirement for consumption of its agricultural good whereas Foreign does not have such a constraint. This realistic asymmetric structure allows us to capture cleanly the competition between the two forces identified by the literature.[2]

Our main finding is that, higher Home agricultural productivity has ambiguous effects on Home industrialization and its transitional growth rate, while it hastens Foreign industrialization and raises its transitional growth rate. On the other hand, higher Foreign agricultural productivity hastens Home industrialization and raises its transitional growth rate, while it delays Foreign industrialization and lowers its transitional growth rate.

The mechanism driving the ambiguous results on industrialization and transitional growth is the competition between the change in the Home consumption pattern, which is governed by the subsistence requirement for its agricultural good, and the change in the degree of specialization due to international trade. The consumption pattern force reallocates labor from farm to factory. Specifically, as Home agricultural productivity rises, the amount of agricultural labor required to satisfy the subsistence requirement declines. However, the specialization force reallocates labor from factory to farm. Since domestic and foreign agricultural goods are

[1] The insight here is that households increase their demand for industrial goods as the agricultural subsistence constraint becomes easier to satisfy; see Chu, Peretto and Wang (2022) for a recent study and a discussion of earlier studies in this literature.

[2] For example, if we let Foreign be the US and Home be either China or Japan, as we do in our quantitative analysis, this characterization is reasonable because the US imports a negligible amount of agricultural products from China and Japan, while China and Japan are among the largest importers of US agricultural goods. Similarly, ruling out the agricultural subsistence requirement is reasonable for a rich country like the US since it no longer affects household behavior.

substitutes, demand for the agricultural good is elastic.[3] As Home agricultural productivity rises and the price of its agricultural good falls, the quantity demanded increases by more than one for one, which in turn requires more agricultural labor. Consequently, an improvement in Home's agricultural productivity generates opposing effects: the consumption pattern force releases labor from farm to factory, while the specialization force draws labor back into farm, producing an ambiguous effect. In contrast, Foreign unambiguously reallocates labor from agriculture to industry, since the demand for its agricultural good declines. To test this insight, we look at two special cases that isolate the two forces.

In the first special case, we shut down the Home subsistence requirement for consumption of its agricultural good. The effects are as in the general case with the key difference that the effects of higher Home agricultural productivity are no longer ambiguous. An improvement in agricultural productivity delays industrial development because the economy specializes further in agricultural production and thus reallocates labor from factory to farm. This scenario is consistent with Mokyr (1976), Field (1978) and Wright (1979).[4] Furthermore, an improvement in the agricultural productivity of its trading partner has the opposite effects on the domestic economy: it hastens domestic industrial development and raises the domestic transitional growth rate. The insight here is that the assumption of no Home subsistence requirement for its agricultural good shuts down one of the two forces highlighted above, the change in the consumption pattern of the Home households, which in this scenario plays no role.

In the second special case, we shut down international agricultural trade. In this scenario, higher Home agricultural productivity hastens Home industrialization and raises its transitional growth rate. The reason is that the dominant force is the change in the consumption pattern of the Home households, which results in a reallocation of labor from farm to factory, because the specialization force is mitigated when there is no agricultural trade.[5] This scenario is consistent with Nurkse (1953) and Schultz (1953). Furthermore, this property yields that the Foreign agricultural productivity has no effects on industrialization in Home and Foreign.

An important aspect of our analysis is that the economy's steady-state growth rate is always independent of the level of agricultural productivity due to the scale-invariance of our Schumpeterian growth model with endogenous market structure. This property highlights the importance of considering the entire transition dynamics of our trading economies: to see the growth effects of agricultural productivity, we must look at the transitional growth rate because the steady-state growth rate does not respond to factors that operate through the scale of economic activity.

To set the stage for our theoretical analysis, we examine the two theoretical predictions in the literature using cross-country panel data. We find that agricultural productivity has a direct positive effect on economic growth and an indirect negative effect via agricultural trade.

[3] As the data suggests, the elasticity of substitution between domestic and foreign agricultural goods is greater than one, which implies that domestic and imported agricultural goods are substitutes; see Feenstra *et al.* (2018) and Bajzik *et al.* (2020). We also consider the case where domestic and foreign agricultural goods are complements in Appendix D.

[4] See Matsuyama (1992) for a theoretical formalization of this idea. Under the zero subsistence requirement, our model reproduces the negative link between agricultural productivity and industrialization emphasized in Matsuyama (1992). However, in the general version of our model, the overall effect of agricultural productivity can be either positive or negative.

[5] Chu, Peretto and Wang (2022) obtains this effect in a closed-economy Schumpeterian growth model. Our contribution here is to examine when this positive effect also shows up in an open-economy setting.

Therefore, the overall effect of agricultural productivity on economic growth is ambiguous and becomes negative as the country's reliance on agricultural imports becomes sufficiently high. In other words, when the country's agricultural imports are small (large) relative to its own agricultural production, higher domestic agricultural productivity stimulates (stifles) industrial development and economic growth as our theory predicts. In addition, we find that the magnitude of effects of agricultural productivity weakens as the economy develops and eventually disappears, also as our theory predicts.

To shed further light on the mechanism driving the theory, we calibrate our model to data for the China-US and Japan-US pairs. In the China-US case, agricultural productivity in China has an overall positive effect on its economic growth due to its relatively low reliance on agricultural imports. In particular, since agricultural consumption relies mostly on domestic agricultural production, the specialization force is weak and the consumption pattern force dominates. An increase in domestic agricultural productivity alleviates China's subsistence constraint on agricultural consumption. As a result, labor can be reallocated from the agricultural sector to the industrial sector. In the Japan-US case, in contrast, Japan's agricultural productivity has an overall negative effect on its economic growth, because Japan relies heavily on agricultural imports, making the specialization force relatively strong. Specifically, higher agricultural productivity causes the Japanese economy to engage in agricultural import substitution and thereby reallocate labor from industrial production to agricultural production. Quantitatively, changes in domestic agricultural productivity explain about 30 percent of the increase in China's technological growth rate and 15 percent of the decline in Japan's technological growth rate. In addition, changes in agricultural productivity in China or Japan also contribute significantly to the increase in the growth rate of technology in the US.

This study contributes to the literature on innovation-led economic growth. Romer (1990) develops the seminal R&D-based growth model of variety expansion. Aghion and Howitt (1992) develop the creative-destruction Schumpeterian growth model of quality improvement.[6] Subsequent studies develop the Schumpeterian growth model with endogenous market structure, which incorporates both variety-expanding and quality-improving innovation; see Peretto (1994, 1998, 1999), Smulders (1994), Smulders and van de Klundert (1995), Dinopoulos and Thompson (1998) and Howitt (1999).[7] Many of these models feature firms that do in-house R&D to fuel incremental innovation (i.e., creative accumulation); the others feature firms that do not do in-house R&D and wait to be replaced by outside challengers (i.e., creative destruction). Garcia-Macia *et al.* (2019) provide the most recent empirical evidence that economic growth comes mostly from creative accumulation rather than creative destruction. We contribute to this literature by developing an open-economy creative-accumulation Schumpeterian growth model with an agricultural sector that produces tractable transitional dynamics featuring an endogenous takeoff. We then use the model to explore the effects of agricultural productivity on the endogenous transition from pre-industrial stagnation to innovation-driven growth of economies that engage in industrial and agricultural trade.

This study also contributes to the literature on agricultural productivity, industrialization and economic development. Early studies by Nurkse (1953), Schultz (1953) and Rostow (1959) argue that agricultural productivity growth releases labor from agriculture to industry and

[6] See also the early studies by Grossman and Helpman (1991) and Segerstrom *et al.* (1990).

[7] See Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008) and Ang and Madsen (2011) for empirical evidence that supports the Schumpeterian growth model with both dimensions of innovation.

serves as an essential engine of industrialization and economic development.[8] Johnston and Mellor (1961), Mellor (1995) and Johnson (1997) echo this view. Subsequent studies formalize it; see for example, Ranis and Fei (1961) for an extended Lewis model with an institutional wage and Murphy *et al.* (1989), Kogel and Prskawetz (2001) and Restuccia *et al.* (2008) for a two-sector general equilibrium model. Empirical studies supportive of these theoretical developments are Tiffin and Irz (2006), Gollin *et al.* (2011) and Cao and Birchenall (2013), McArthur and McCord (2017). On the other hand, Mokyr (1976), Field (1978) and Wright (1979) stress the importance of international trade and, in contrast to the view just discussed, argue that high agricultural productivity gives rise to specialization in agriculture and delays industrialization. Subsequent studies by Matsuyama (1992), Duranton (1998) and Chesnokova (2007) formalize this idea and find that higher agricultural productivity triggers early industrialization in a closed economy but delays industrialization in an open economy. Foster and Rosenzweig (2004, 2008) provide empirical evidence for this negative relationship between agricultural productivity and economic growth;[9] see also Gollin (2010) for a thorough review of both theoretical and empirical studies in this literature. Despite the richness of the theoretical literature that studies the role of agricultural productivity in structural transformation and economic development driven by capital accumulation, relatively few studies examine its effects on innovation-driven growth. We contribute to this vast literature by developing an open-economy Schumpeterian growth model that allows us to explore the effects of agricultural productivity on innovation-driven growth in the presence of international trade in agricultural goods. The goal is to sort out the relative contribution of the contrasting forces at play.

This study also relates to the literature on structural transformation. For example, Matsuyama (1992), Echevarria (1995, 1997), Laitner (2000), Kongsamut *et al.* (2001), Lucas (2004), Gollin *et al.* (2002, 2007) and Ngai and Pissarides (2007) incorporate an agriculture sector into growth models to explore the structural transformation from agriculture to industry.[10] However, these studies do not consider the role of agricultural productivity on the innovation-driven takeoff of the economy.

Finally, this study contributes to the literature on endogenous takeoff and economic growth. The seminal study by Galor and Weil (2000) develops Unified Growth Theory (UGT), which captures the process of transformation from a Malthusian agricultural economy to a modern industrial economy in a single analytical framework. Subsequent studies by Galor and Moav (2002), Galor and Mountford (2008), Galor *et al.* (2009) and Ashraf and Galor (2011) examine the role of different prehistorical and historical characteristics and provide supportive empirical evidence for UGT.[11] In a related literature, Peretto (2015) develops a closed-economy Schumpeterian growth model with endogenous takeoff to capture the endogenous transition

[8]In the seminal study by Lewis (1955), the agricultural sector is characterized by labor surplus and disguised unemployment. Also, Krugman (1987) and Lucas (1988) argue that the manufacturing sector is characterized by economies of scale and human capital accumulation.

[9]Bravo-Ortega and Lederman (2005) find a positive effect of agricultural productivity on growth in non-agricultural sectors in developing countries, but this effect is negative in developed countries. See also Bustos *et al.* (2022) who show that high agricultural productivity causes structural transformation but not innovation in Brazil.

[10]In a related literature, Leukhina and Turnovsky (2016) explore how population growth affects structural transformation, whereas Huneeus and Rogerson (2024) examine the effects of agricultural productivity on premature deindustrialization.

[11]See Galor (2005, 2011) for a comprehensive review of UGT.

from pre-industrial stagnation to innovation-driven growth.[12] Chu, Peretto and Wang (2022) develop a Schumpeterian growth model with an agricultural sector to explore how agricultural productivity affects the transition of an economy from pre-industrial stagnation to innovation-driven growth in a closed economy. Chu, Peretto and Xu (2023) develop a small open economy version of the Schumpeterian growth model in Peretto (2015) to explore export-led takeoff and innovation-driven growth. This study introduces agricultural trade to a two-country version of the Schumpeterian growth model in Chu, Peretto and Xu (2023) in order to explore the novel implications of agricultural trade on the effects of agricultural productivity on industrial take-off. Therefore, we contribute to this literature by extending the Schumpeterian growth model with endogenous takeoff to the case of a world general equilibrium featuring two countries that trade both industrial and agricultural goods.

The rest of this study is organized as follows. Section 2 documents some stylized facts using cross-country panel data. Section 3 presents our open-economy Schumpeterian growth model with an agricultural sector. Section 4 characterizes the effects of agricultural productivity improvement. Section 5 calibrates the model and investigates the quantitative effects of changes in agricultural productivity. Section 6 presents two extensions. Section 7 concludes.

2 Stylized facts

In this section, we use cross-country data to establish some key facts about the relationship between agricultural productivity, agricultural trade and the transitional rate of economic growth. We use the following specification:

$$y_{jt} = \kappa_1 A_{jt} + \kappa_2 A_{jt} \times trade_{jt} + \kappa_3 trade_{jt} + \Gamma\Phi_{jt} + \zeta_j + \zeta_t + \varepsilon_{jt},$$

where y_{jt} is a proxy for industrialization or economic growth in country j at time t, measured by the log level of real GDP per capita, or the log level of non-agricultural real GDP per capita, or the log level of total factor productivity (TFP) index. A_{jt} is agricultural productivity in country j at time t, for which we follow McArthur and McCord (2007) to use the log level of cereal yields per hectare as a proxy.[13] $trade_{jt}$ denotes the initial degree of reliance on agricultural imports in country j at time t, measured by the cereal import dependency ratio.[14] Given that cyclical fluctuations in annual data may bias the estimation, we consider five years as a period. Our theory predicts that $\kappa_1 > 0$ and $\kappa_2 < 0$. In other words, high agricultural productivity has a positive effect on industrialization and economic growth, but this positive effect weakens and may become negative when an economy relies heavily on agricultural imports.[15] Φ_{jt} denotes

[12] See also the subsequent studies by Iacopetta and Peretto (2021), Chu, Fan and Wang (2020), Chu, Kou and Wang (2020) and Chu, Furukawa and Wang (2022) for different mechanisms that trigger endogenous takeoff in this framework.

[13] From the FAO's definition, cereals include rice, wheat, maize, barley, oats, millet, and sorghum, etc. Moreover, cereals serve as the primary source of calories and plant protein in global diet; see Poutanen *et al.* (2022).

[14] According to FAO (2012), the import dependency ratio is defined as $(imports)/(production + imports - exports)$. The cereal import dependency ratio serves as a common measure of agricultural import reliance; see for example, Clapp (2017) and FAO (2022).

[15] Our theory features a positive growth effect governed by the subsistence requirement and a negative growth effect operating through agricultural international trade. Without agricultural imports, the growth effect of agricultural technology through agricultural international trade disappears.

the following set of control variables: the log level of capital stock, government spending as a share of GDP, the depreciation rate of capital stock, and the real interest rate. The variables ζ_j and ζ_t denote country and time fixed effects, respectively. Finally, ε_{jt} is the error term.

After merging data from the Food and Agricultural Organization (FAO), the Penn World Tables, the U.S. Department of Agriculture and the World Bank Data, we have a sample of up to 788 observations covering 149 countries for 1991-2020. Table A1 in Appendix A provides the summary statistics.

Table 1: Relationship between agricultural productivity, trade and economic growth

	log GDP per capita		log non-agri GDP per capita		log TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{jt}	0.443***	0.212***	0.453***	0.224***	0.246***	0.265***
	(0.087)	(0.078)	(0.097)	(0.084)	(0.073)	(0.078)
$A_{jt} \times trade_{jt}$	-0.441***	-0.358***	-0.513***	-0.426***	-0.415***	-0.398***
	(0.116)	(0.114)	(0.126)	(0.125)	(0.112)	(0.115)
$trade_{jt}$	4.371***	3.527***	5.050***	4.199***	3.984***	3.856***
	(1.138)	(1.122)	(1.234)	(1.227)	(1.102)	(1.144)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Period fixed effects	✓	✓	✓	✓	✓	✓
Observations	788	608	775	601	540	540
R^2	0.9878	0.9906	0.9879	0.9909	0.6595	0.6881

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses, clustered at the country level.*

Table 1 reports the estimation results.[16] The dependent variable is the log of real GDP per capita in columns (1)-(2), the log of non-agricultural real GDP per capita in columns (3)-(4), and the log of TFP index in columns (5)-(6).[17] In all columns, the coefficient κ_1 on agricultural productivity is significantly positive and the coefficient κ_2 on the interaction term between agricultural productivity and reliance on agricultural imports is significantly negative.[18] Taking column (6) as an example, the coefficient on agricultural productivity is 0.265 and the coefficient on the interaction with cereal dependency ratio is -0.398, both significant at the 1% level. Specifically, for the economy with the minimal cereal import dependency ratio, a 1% increase in agricultural productivity is associated with an increase of 0.265% (= $(0.265 - 0 \times 0.398)$%) in economic growth, which is statistically significant at the 1% level. For

[16] We also consider a sub-sample that consists only of African and Asian countries, among which most countries are still in the transition period. Our empirical results remain robust; see Table A2. In addition, the results do not change substantially even when we further exclude developed countries in this sub-sample (i.e., Japan, Korea and Singapore); results are available upon request.

[17] Given the potential persistence of our dependent variable, we also consider its lag to prevent omitted variable bias and isolate the effect of agricultural productivity. The results remain robust; see Table A3.

[18] As alternative proxies, we use the agricultural TFP index for agricultural productivity and the ratio of agricultural imports value to domestic agricultural GDP for the reliance on agricultural imports. The results remain robust; see Table A4.

the economy with the average cereal import dependency ratio, a 1% increase in agricultural productivity is associated with an increase of 0.126% ($\approx (0.265 - 0.350 \times 0.398)$%) in economic growth, which is statistically significant at the 1% level. For the economy with the maximal cereal import dependency ratio, a 1% increase in agricultural productivity is associated with a decrease of 0.133% ($= (0.265 - 1 \times 0.398)$%) in economic growth, which is statistically significant at the 1% level. These results show that the positive relationship between domestic agricultural productivity and economic growth weakens and may even become negative as a country relies more heavily on agricultural imports.[19]

To alleviate any potential endogeneity concern, we follow Bartik (1991), Nunn and Qian (2011) and Goldsmith-Pinkham *et al.* (2020) to construct an instrument for agricultural productivity using soil quality and precipitation, which are typically not influenced by economic outcomes.[20] Specifically, we construct a Bartik-style instrument by interacting time-invariant soil quality with annual precipitation shocks to predict cereal yields.[21] The relationship between our geo-climatic-based measure and the actual cereal yields is relatively strong with an $F-$statistic of 35.55; see Table A5. The predicted values in this regression capture the component of agricultural productivity that is explained by geo-climatic conditions (i.e., soil quality and precipitation), thereby removing the influence of human intervention and economic development. Therefore, using this geo-climate-based series of cereal yields as an instrument for agricultural productivity mitigates the potential concern that agricultural productivity could be correlated with economic growth. Table A6 in Appendix A reports the IV estimation results. In particular, the coefficient κ_1 on agricultural productivity remains statistically significant and positive, while the coefficient κ_2 on the interaction term remains statistically significant and negative. The results remain consistent with the baseline regression results.[22]

Our theory also predicts that the effects of agricultural productivity eventually disappear as the economy reaches its balanced growth path. We employ panel quantile regression to test this prediction. The specification is as follows

$$Q_\tau (y_{jt}) = \kappa_{\tau,1} A_{jt} + \kappa_{\tau,2} A_{jt} \times trade_{jt} + \kappa_{\tau,3} trade_{jt} + \Gamma_\tau \Phi_{jt} + \zeta_j + \zeta_t,$$

where $Q_\tau(y_{jt})$ denotes τth conditional quantile of the dependent variable y_{jt}. Table 2 reports the quantile regression results. The dependent variable is the log level of real GDP per capita in Panel A, the log level of non-agricultural real GDP per capita in Panel B, and the log level of TFP index in Panel C. In Panels A-C, the coefficient on agricultural productivity κ_1, which captures its growth effect, shows a decreasing trend when the economy moves to

[19]Our empirical result also indicates that the overall effect of agricultural productivity on TFP growth is significantly positive in China but significantly negative in Japan, which is consistent with our quantitative analysis.

[20]See Ray *et al.* (2015) for a discussion of the influence of precipitation on crop yields. See also Jayachandran (2006) who uses precipitation as an instrument for agricultural productivity.

[21]Data source: Michalopoulos (2012) and *ERA5* dataset provided by European Centre for Medium-Range Weather Forecasts (ECMWF).

[22]To mitigate concerns that our proxy for the reliance of agricultural imports might be endogenous, we follow Acemoglu and Restrepo (2020) to construct a Bartik-style measure for agricultural import reliance. Specifically, we interact each country's initial import dependency ratio in 1991 with relative change in the international cereal price from the previous year to the current year. Since the exposure term is fixed at its initial value and the price shocks are determined in the international market which are orthogonal to individual countries' economic outcome, this Bartik measure provides plausibly exogenous variation in agricultural import reliance. Table A7 reports the estimated results using this Bartik-style measure.

higher quantiles. Moreover, coefficients κ_1 and κ_2 become statistically insignificant when the economy is at the 99th percentile. These results suggest that the magnitude of effects of agricultural productivity weakens as the economy develops, and eventually disappears as the economy converges to its balanced growth path.

Table 2: Panel quantile regression

Percentile	10th (1)	25th (2)	50th (3)	75th (4)	90th (5)	99th (6)
Panel A: dependent variable - log GDP per capita						
A_{jt}	0.261***	0.243***	0.214***	0.182**	0.166*	0.012
	(0.101)	(0.090)	(0.080)	(0.081)	(0.088)	(0.625)
$A_{jt} \times trade_{jt}$	-0.326**	-0.338***	-0.357***	-0.378***	-0.388***	-0.489
	(0.134)	(0.125)	(0.117)	(0.119)	(0.124)	(0.504)
Panel B: dependent variable - log non-agri GDP per capita						
A_{jt}	0.280**	0.258**	0.226**	0.190**	0.173*	0.019
	(0.114)	(0.101)	(0.089)	(0.088)	(0.095)	(0.508)
$A_{jt} \times trade_{jt}$	-0.403***	-0.412***	-0.425***	-0.439***	-0.446***	-0.507
	(0.145)	(0.135)	(0.128)	(0.130)	(0.136)	(0.709)
Panel C: dependent variable - log TFP						
A_{jt}	0.396***	0.341***	0.263***	0.201***	0.154**	-0.313
	(0.129)	(0.102)	(0.078)	(0.066)	(0.062)	(0.642)
$A_{jt} \times trade_{jt}$	-0.471***	-0.440***	-0.397***	-0.362***	-0.335***	-0.072
	(0.159)	(0.140)	(0.119)	(0.106)	(0.099)	(0.406)
Control variables	✓	✓	✓	✓	✓	✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Period fixed effects	✓	✓	✓	✓	✓	✓

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Bootstrapped standard errors (with 500 replications) are reported in parentheses, clustered at the country level.*

3 A Schumpeterian model with agricultural trade

In this section, we develop a two-country Schumpeterian growth model to explore the role of agricultural productivity in driving the endogenous takeoff of the economy and its convergence to scale-invariant steady-state growth driven by both variety expansion and quality improvement. The model is based on Peretto (2015) but is also inspired by Peretto and Valente (2011), who develop the first two-country, world general equilibrium model of endogenous innovation with asymmetric trade due to different endowments of natural resources. Chu, Peretto and Wang (2022) introduce an agricultural sector to the model in Peretto (2015), obtaining a mechanism through which agricultural productivity affects endogenous takeoff in a closed economy. By converting the closed-economy model into a two-country model, we incorporate international agricultural trade as a novel element in order to shed light on the relationship between agricultural productivity and innovation-driven growth, via international trade in both agricultural and industrial goods.

3.1 Households

There are two countries: Home, denoted h, and Foreign, denoted f. To ensure the existence of a balanced-growth path in our two-country world-economy model, we assume that the two countries have the same population growth rate, denoted as $\lambda > 0$. With the same population growth rate in the two countries, the population ratio remains constant at the value $L_t^h/L_t^f = L_0^h/L_0^f$, where L_0^h and L_0^f are the initial populations of Home and Foreign, respectively.

The representative household in country $j \in \{h, f\}$ has preferences

$$U^j = \int_0^\infty e^{-(\rho^j-\lambda)t} \left\{ \ln c_t^j + \psi^j \ln \iota_t^j + \gamma^j \ln \left[\delta^j (q_t^j - \eta^j)^{\frac{\epsilon-1}{\epsilon}} + (1-\delta^j)(m_t^j)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \right\} dt, \tag{1}$$

where c_t^j, ι_t^j, q_t^j and m_t^j denote, respectively, consumption per capita of the domestic industrial good, of the imported industrial good, of the domestic agricultural good and of the imported agricultural good. The parameter $\rho^j > \lambda$ is the subjective discount rate. The parameters $\psi^j > 0$ and $\gamma^j > 0$ regulate the contribution to flow utility of the imported industrial good and of the agricultural goods. The parameter $\delta^j \in (0, 1]$ regulates the importance of the domestic agricultural good relative to the imported agricultural good and $\eta^j > 0$ is the subsistence requirement for consumption of the domestic agricultural good.[23] Finally, $\epsilon \in (1, \infty)$ is the elasticity of substitution between domestic and foreign agricultural goods.[24]

The asset-accumulation equation in country j is given by

$$\dot{a}_t^j = (r_t^j - \lambda)a_t^j + w_t^j - p_{Y,t}^j c_t^j - p_{Y,t}^{-j} \iota_t^j - p_{A,t}^j q_t^j - p_{A,t}^{-j} m_t^j, \tag{2}$$

where the superscript $-j$ denotes a country other than country j. a_t^j is the value of asset per capita and r_t^j is the interest rate. Each household member supplies inelastically one unit of labor to earn the wage rate w_t^j. In addition, $p_{Y,t}^j$ and $p_{Y,t}^{-j}$ are, respectively, the price of domestic industrial good and of imported industrial good. Similarly, the prices of domestic and imported agricultural goods are denoted by $p_{A,t}^j$ and $p_{A,t}^{-j}$, respectively.

The household's dynamic optimization in country j yields the consumption Euler equation

$$\frac{\dot{c}_t^j}{c_t^j} = r_t^j - \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j} - \rho^j \tag{3}$$

and the expenditure on the imported industrial good

$$p_{Y,t}^{-j} \iota_t^j = \psi^j p_{Y,t}^j c_t^j. \tag{4}$$

[23] The subsistence requirement is a key feature of agricultural consumption. Gollin *et al.* (2007) and Tombe (2015) show that countries tend to be self-sufficient in agriculture, and their subsistence requirements are largely satisfied with domestically produced cereals rather than imported agricultural good.

[24] Feenstra *et al.* (2018) and Bajzik *et al.* (2020) show that the elasticity of substitution between domestic and foreign agricultural goods is greater than 1. We also consider the case $\epsilon \in (0, 1)$ in Appendix D.

Up to this point, the model treats the two countries symmetrically.

To obtain a sharp characterization of the role of agricultural productivity, we set up an asymmetric agricultural trade structure. Specifically, the Home representative household consumes both domestic and foreign agricultural goods. The Foreign representative household, instead, consumes only the domestic agricultural good.[25] Technically, we set $\delta^f = 1$, which yields $m_t^f = 0$. Moreover, because the model does not have a balanced growth path if we allow for a Foreign subsistence requirement for its own agricultural good, we set $\eta^f = 0$.

With this structure, the Home household's dynamic optimization yields the expenditure functions for domestic and foreign agricultural goods:

$$p_{A,t}^h(q_t^h - \eta^h) = \frac{\delta^h \gamma^h p_{Y,t}^h c_t^h}{\delta^h + (1-\delta^h)/\left(\frac{q_t^h - \eta^h}{m_t^h}\right)^{\frac{\epsilon-1}{\epsilon}}}; \tag{5}$$

$$p_{A,t}^f m_t^h = \frac{(1-\delta^h)\gamma^h p_{Y,t}^h c_t^h}{\delta^h \left(\frac{q_t^h - \eta^h}{m_t^h}\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}. \tag{6}$$

Taking the ratio of these expressions, we obtain

$$b_t^h \equiv \frac{q_t^h - \eta^h}{m_t^h} = \left(\frac{\delta^h}{1-\delta^h}\frac{p_{A,t}^f}{p_{A,t}^h}\right)^{\epsilon}. \tag{7}$$

This variable b_t^h, which captures the ratio of domestic agricultural consumption to imported agricultural consumption, plays a crucial role in our analysis. Therefore, we give it a name and a symbol: we call b_t^h the *agricultural consumption ratio*. The dynamic optimization of the Foreign representative household yields the expenditure function for its own agricultural good

$$p_{A,t}^f q_t^f = \gamma^f p_{Y,t}^f c_t^f. \tag{8}$$

The detailed derivation of these relations is in Appendix C.

3.2 Agriculture

We follow Lagakos and Waugh (2013) and assume a perfectly competitive market and constant returns to scale in agriculture. Specifically, agricultural production function in country j is given by

$$Q_t^j = A^j L_{A,t}^j = A^j l_{A,t}^j L_t^j, \tag{9}$$

where Q_t^j is agricultural output, $L_{A,t}^j$ and $l_{A,t}^j$ are, respectively, the agricultural labor input and the agricultural labor share, and the parameter $A^j > 0$ denotes agricultural productivity. We set $A^h > \eta^h$ to ensure that the Home economy is viable in the sense that it satisfies its agricultural subsistence constraint.[26]

[25] See the discussion in footnote 3.

[26] To focus on the effects of agricultural technology instead of its causes, we do not model agricultural technological improvement as an endogenous progress; instead, we consider exogenous changes in the level of agricultural technology. In Section 6.2, we extend our model by introducing a spillover effect from the industrial sector to the agricultural sector.

Profit maximization yields the price of the agricultural good

$$p_{A,t}^{j} = \frac{w_t^j}{A^j}. \tag{10}$$

The agricultural market-clearing condition in country j is

$$Q_t^j = q_t^j L_t^j + m_t^{-j} L_t^{-j}, \tag{11}$$

where the superscript $-j$ denotes a country other than country j. In interpreting this condition, recall that by construction $m_t^f = 0$.

3.3 Industrial good

In country j, competitive firms produce the industrial good with the technology

$$Y_t^j = \int_0^{N_t^j} [X_t^j(i)]^{\theta^j} \left\{ [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} \right\}^{1-\theta^j} di, \tag{12}$$

where the parameter $\theta^j \in (0,1)$ determines labor intensity $1-\theta^j$ in industrial production. N_t^j is the variety of intermediate goods, and $X_t^j(i)$ is the quantity of intermediate good i. Intermediate good i has quality $Z_t^j(i)$. The average quality across intermediate goods is $Z_t^j \equiv \int_0^{N_t^j} Z_t^j(i)di/N_t^j$. The parameter $\alpha^j \in (0,1)$ regulates the importance of own quality relative to technology spillovers. $L_{Y,t}^j = l_{Y,t}^j L_t^j$ is the industrial labor input, where $l_{Y,t}^j$ is the industrial labor share. The parameter $\sigma^j \in (0,1)$ measures the degree of love of variety in industrial production.

Let $p_{X,t}^j(i)$ be the price of intermediate good i. Profit maximization yields the demand function for intermediate goods

$$X_t^j(i) = \left[\frac{\theta^j}{p_{X,t}^j(i)/p_{Y,t}^j} \right]^{\frac{1}{1-\theta^j}} [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} \quad \text{for } i \in \left[0, N_t^j\right] \tag{13}$$

and the expenditure rules:

$$w_t^j L_{Y,t}^j = (1-\theta^j) p_{Y,t}^j Y_t^j; \tag{14}$$

$$\int_0^{N_t^j} p_{X,t}^j(i) X_t^j(i) di = \theta^j p_{Y,t}^j Y_t^j. \tag{15}$$

The second equation yields our measure of the size of the market for intermediate goods.

3.4 Intermediate goods and in-house R&D

In country j, the typical monopolistic firm uses $X_t^j(i)$ units of the domestic industrial good to produce $X^j(i)$ units of intermediate good i, and incurs a fixed operating cost of $\phi^j [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j}$ units of the domestic industrial good. The profit before R&D is

$$\Pi_t^j(i) = p_{X,t}^j(i) X_t^j(i) - p_{Y,t}^j X_t^j(i) - p_{Y,t}^j \phi^j [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j}. \tag{16}$$

The firm also invests $R_t^j(i)$ units of the domestic industrial good to obtain quality improvement

$$\dot{Z}_t^j(i) = R_t^j(i). \tag{17}$$

Given initial condition $Z_0^j(i)$, the firm maximizes its value,

$$V_t^j(i) = \int_t^\infty \exp\left(-\int_t^s r_u du\right)[\Pi_s^j(i) - p_{Y,s}^j R_s^j(i)]ds, \tag{18}$$

subject to the demand function (13), the profit equation (16), the in-house R&D technology (17); see Appendix C for the solution to this dynamic optimization problem.

We assume that firms start with identical initial conditions, i.e., $Z_0^j(i) = Z_0^j$ for $i \in [0, N_t^j]$. It follows that firms solve identical problems and thus charge identical prices and invest at the same rate. This yields a symmetric equilibrium where $p_{X,t}^j(i) = p_{X,t}^j$, $X_t^j(i) = X_t^j$, $\Pi_t^j(i) = \Pi_t^j$ and $Z_t^j(i) = Z_t^j$ for $i \in [0, N_t^j]$. In particular, firm i sets

$$p_{X,t}^j(i) = \min\left\{\mu^j p_{Y,t}^j, \frac{1}{\theta^j} p_{Y,t}^j\right\} = \mu^j p_{Y,t}^j, \tag{19}$$

where $\mu^j \in (1, 1/\theta)$ represents the number of units of the domestic industrial good that fringe competitive firms require to produce one unit of intermediate good i with the same quality $Z_t(i)$. Therefore, firm i sets the monopolistic price as $\mu^j p_{Y,t}^j$ to drive such fringe firms out of the market. Moreover, the R&D decision of firm i yields the rate of return to quality improvement

$$r_t^j = \alpha^j \left[(\mu^j - 1)\frac{X_t^j}{Z_t^j} - \phi^j\right] + \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j}, \tag{20}$$

where X_t^j/Z_t^j is the quality-adjusted size of the firm i, defined as the quantity sold per unit of quality. Substituting the price $p_{X,t}^j(i) = \mu^j p_{Y,t}^j$ into (13) yields

$$\frac{X_t^j}{Z_t^j} = \left(\frac{\theta^j}{\mu^j}\right)^{\frac{1}{1-\theta^j}} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} = \left(\frac{\theta^j}{\mu^j}\right)^{\frac{1}{1-\theta^j}} \frac{L_t^j l_{Y,t}^j}{(N_t^j)^{1-\sigma^j}}. \tag{21}$$

This expression contains the two key state variables of the model, namely, the endogenous mass of firms, N_t^j, and the exogenous population, L_t^j.

To characterize the dynamics of the model analytically, we define the composite state variable

$$x_t^j \equiv \left(\frac{\theta^j}{\mu^j}\right)^{\frac{1}{1-\theta^j}} \frac{L_t^j}{(N_t^j)^{1-\sigma^j}}. \tag{22}$$

In this notation, the rate of return to quality-improving innovation is

$$r_t^j = \alpha^j[(\mu^j - 1)x_t^j l_{Y,t}^j - \phi^j] + \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j}, \tag{23}$$

where $x_t^j l_{Y,t}^j$ is the quality-adjusted size of the firm. We shall use the shorthand *firm size* for this variable when confusion does not arise.

3.5 Entrants

In pursuit of monopolistic profit, new firms have incentives to enter the market, providing new differentiated intermediate goods of average quality. Entering the market requires payment of a sunk entry cost (for setting up equipment and plant). In country j, entry is positive when the free-entry condition

$$V_t^j = \beta^j p_{Y,t}^j X_t^j \tag{24}$$

holds, where $\beta^j > 0$ is an entry-cost parameter.

Given the intermediate industry equilibrium is symmetric, we differentiate the firm-value equation (18) with respect to time to obtain

$$r_t^j = \frac{\Pi_t^j - p_{Y,t}^j R_t^j}{V_t^j} + \frac{\dot{V}_t^j}{V_t^j}. \tag{25}$$

This is the standard asset-pricing equation that defines the rate of return to owning equity in a firm. Substituting the profit equation (16), the R&D technology (17), the monopolistic price (19), the expression for quality-adjusted firm size (21), the definition of x_t^j (22) and the free entry condition (24) into the asset pricing equation (25) yields the rate of return to entry or, equivalently, to firm ownership:

$$r_t^j = \frac{(\mu^j - 1)x_t^j l_{Y,t}^j - \phi^j - z_t^j}{\beta^j x_t^j l_{Y,t}^j} + \frac{\dot{x}_t^j}{x_t^j} + \frac{\dot{l}_{Y,t}^j}{l_{Y,t}^j} + z_t^j + \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j}, \tag{26}$$

where $z_t^j \equiv \dot{Z}_t^j / Z_t^j$ is the growth rate of quality.

3.6 International trade

In our model, the Home representative household consumes domestic and imported agricultural goods as well as domestic and imported industrial goods. The Foreign representative household also consumes domestic and imported industrial goods but consumes only its domestic agricultural good. Therefore, the balanced-trade condition is

$$p_{Y,t}^h \iota_t^f L_t^f = p_{Y,t}^f \iota_t^h L_t^h + p_{A,t}^f m_t^h L_t^h. \tag{27}$$

3.7 Equilibrium

The equilibrium is a time path of allocations $\{c_t^j, \iota_t^j, q_t^j, m_t^j, l_{Y,t}^j, l_{A,t}^j, X_t^j(i), R_t^j(i)\}$ and a time path of prices $\{w_t^j, r_t^j, p_{Y,t}^j, p_{Y,t}^{-j}, p_{X,t}^j(i), p_{A,t}^j, p_{A,t}^{-j}, V_t^j(i)\}$ in country j such that:

- households choose $\{c_t^j, \iota_t^j, q_t^j, m_t^j\}$ to maximize utility taking $\{w_t^j, r_t^j, p_{Y,t}^j, p_{Y,t}^{-j}, p_{A,t}^j, p_{A,t}^{-j}\}$ as given;

- competitive agricultural firms choose agricultural labor input $L_{A,t}^j$ to maximize profit taking $\{w_t^j, p_{A,t}^j\}$ as given;

- competitive industrial firms choose factor inputs $\{L_{Y,t}^j, X_t^j(i)\}$ to maximize profit taking $\{w_t^j, p_{Y,t}^j, p_{X,t}^j(i)\}$ as given;

- monopolistic intermediate firms choose $\{p^j_{X,t}(i), R^j_t(i)\}$ to maximize their value $V^j_t(i)$ taking $\{r^j_t, p^j_{Y,t}\}$ as given;
- entrants make entry decisions taking $\{V^j_t, p^j_{Y,t}\}$ as given;
- the value of household assets is equal to the value of the monopolistic firms, $a^j_t L^j_t = N^j_t V^j_t$;
- the agricultural good market clears, $Q^j_t = q^j_t L^j_t + m^{-j}_t L^{-j}_t$ (recall that $m^f_t = 0$);
- the labor market clears, $L^j_t = L^j_{Y,t} + L^j_{A,t} = l^j_{Y,t} L^j_t + l^j_{A,t} L^j_t$;
- the industrial good market clears, $Y^j_t = c^j_t L^j_t + N^j_t (X^j_t + \phi^j Z^j_t + R^j_t) + \dot{N}^j_t \beta^j X^j_t + \iota^{-j}_t L^{-j}_t$;
- the balanced-trade condition holds, $p^h_{Y,t} \iota^f_t L^f_t = p^f_{Y,t} \iota^h_t L^h_t + p^f_{A,t} m^h_t L^h_t$.

3.8 Aggregation

We substitute the monopolistic quantity (13) and price (19) into the industrial production function (12) to obtain the equilibrium reduced-form production function in country j

$$Y^j_t = \left(\frac{\theta^j}{\mu^j}\right)^{\frac{\theta^j}{1-\theta^j}} \left(N^j_t\right)^{\sigma^j} Z^j_t L^j_t l^j_{Y,t}. \tag{28}$$

The growth rate of industrial output per capita, defined as $y^j_t \equiv Y^j_t / L^j_t$, is

$$g^j_t \equiv \frac{\dot{y}^j_t}{y^j_t} = \sigma^j n^j_t + z^j_t + \frac{\dot{l}^j_{Y,t}}{l^j_{Y,t}}, \tag{29}$$

where $n^j_t \equiv \dot{N}^j_t / N^j_t$ and $z^j_t \equiv \dot{Z}^j_t / Z^j_t$ are growth rates of variety and quality, respectively.

3.9 Dynamics

Given the definition in equation (22), the law of motion of the state variable x^j_t in country j is

$$\frac{\dot{x}^j_t}{x^j_t} = \lambda - (1 - \sigma^j) n^j_t. \tag{30}$$

We show below that the growth rate of variety, n^j_t, is a monotonically increasing function of x^j_t. Accordingly, x^j_t grows over time and converges to the unique steady state $(x^j)^*$. We also show that there exist two thresholds of the state variable, denoted as x^j_N and x^j_Z, respectively. For $x^j_t < x^j_N$, agents are not willing to finance variety-expanding innovation (i.e., entry) because firm size is too small. Likewise, for $x^j_t < x^j_Z$, agents are not willing to finance quality-improving innovation (i.e., in-house R&D) because firm size is too small.

We choose parameters such that $x^j_N < x^j_Z$ in order to generate a sequence of events that replicates the historical experience of advanced economies. In particular, the economy goes through three phases: the pre-industrial era, characterized by the absence of innovation; the first phase of the industrial era, characterized by variety expansion but no quality improvement; the second phase of the industrial era, characterized by both variety-expanding and quality-improving innovation. The mechanism generating this pattern is as follows. In the pre-industrial era, firm size is insufficiently large to generate positive monopolistic profit and, consequently, insufficiently large to trigger any kind of innovation. As firm size grows due to exogenous population growth, it crosses the threshold for variety-expanding innovation and the economy enters the industrial era. The first phase of the industrial era features the emergence of monopolistic firms taking over existing intermediate goods lines, and the variety-expanding innovation activity of entrants, who invest to capture a share of the market for intermediate goods. As firm size continues to grow, it crosses the threshold for quality-improving innovation. At this point, the economy enters the second phase of the industrial era that features both variety-expanding and quality-improving innovation. Eventually, firm size converges to its steady-state value and the economy settles onto its balanced growth path.

To ensure that the economy converges to the balanced growth path, along which firm size is constant and strictly positive, whereas variety expansion, quality improvement, and per capita output grow at constant positive rates, we impose the parameter condition

$$\beta^j \phi^j > \frac{1}{\alpha^j}\left[\mu^j - 1 - \beta^j\left(\rho^j + \frac{\sigma^j \lambda}{1-\sigma^j}\right)\right] > \mu^j - 1. \tag{31}$$

Under this parameter condition, given the initial firm size $x_0 < x_N$, the economy goes through the three phases before reaching the balanced growth path. The following lemmas describe the key dynamic property of the model: a set of intratemporal relations that determine the fast-adjusting endogenous variables $b_t^h = (q_t^h - \eta^h)/m_t^h$, c_t^j/y_t^j, $l_{Y,t}^j$ and $l_{A,t}^j = 1 - l_{Y,t}^j$ as functions of the model's parameters. Given the constant equilibrium values of these variables, the transitional dynamics of the model are governed by the law of motion of the slow-adjusting state variable x_t^j characterized in equation (30). Under condition (31), this process eventually converges to the steady state $(x^j)^*$.

Lemma 1 *(Intratemporal equilibrium) At any time t, the agricultural consumption ratio b_t^h and the consumption-output ratios $\{c_t^h/y_t^h, c_t^f/y_t^f\}$ jump to the unique and stable steady-state values. In particular, the steady-state values of the agricultural consumption ratio and consumption-output ratios are:*[27]

$$b_t^h = \left(b^h\right)^* = \underset{b_t^h}{\arg\text{solve}}\left\{F(b_t^h;\cdot) = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}\right\},$$

$$\frac{c_t^h}{y_t^h} = \left(\frac{c^h}{y^h}\right)^* = \begin{cases} \dfrac{1-\theta^h}{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}} & 0 \leq x_t^h \leq x_N^h \\ \dfrac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}} & x_N^h < x_t^h < \infty \end{cases},$$

[27]The function $F(b_t^h;\cdot)$ is defined in (B.5) in Appendix B.

$$\frac{c_t^f}{y_t^f} = \left(\frac{c^f}{y^f}\right)^* = \begin{cases} \dfrac{1-\theta^f}{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} & 0 \le x_t^f \le x_N^f \\ \dfrac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} & x_N^f < x_t^f < \infty \end{cases}.$$

where x_N^j is the threshold of firm size for variety-expanding innovation in country j.

Proof. See Appendix B. ■

Lemma 1 shows that the steady-state Home consumption-output ratio is increasing in the steady-state agricultural consumption ratio, while the steady-state Foreign consumption-output ratio is decreasing in it. With the expressions of c_t^h/y_t^h and c_t^f/y_t^f, we derive the industrial labor shares in Lemma 2.

Lemma 2 *(Industrial labor shares) At any time t, the steady-state values of the industrial labor shares $l_{Y,t}^h$ and $l_{Y,t}^f$ are:*

$$l_{Y,t}^h = (l_Y^h)^* = \begin{cases} \dfrac{(1+\psi^h)\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h+\gamma^h)\left[\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}\left(1-\frac{\eta^h}{A^h}\right) & 0 \le x_t^h \le x_N^h \\ \dfrac{(1+\psi^h)\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}{\left[1+\psi^h+\frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}\left(1-\frac{\eta^h}{A^h}\right) & x_N^h < x_t^h < \infty \end{cases}; \tag{32}$$

$$l_{Y,t}^f = (l_Y^f)^* = \begin{cases} \dfrac{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\gamma^f} & 0 \le x_t^f \le x_N^f \\ \dfrac{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} & x_N^f < x_t^f < \infty \end{cases}. \tag{33}$$

Proof. See Appendix C. ■

Lemma 2 suggests that the Home industrial labor share is decreasing in the agricultural consumption ratio, whereas the Foreign industrial labor share is increasing in it.

Lemma 3 *(Comparative statics of agricultural consumption ratio with respect to agricultural productivity) The steady-state value of $\left(b^h\right)^*$ is increasing in the Home agricultural productivity A^h and decreasing in the Foreign agricultural productivity A^f.*

Proof. See Appendix C. ■

According to Lemma 2 and Lemma 3, we summarize the effects of agricultural productivity on industrial labor shares in Proposition 1.

Proposition 1 *(Effects of agricultural productivity on industrial labor shares) The effect of higher Home agricultural productivity on the Home industrial labor share is ambiguous, whereas higher Foreign agricultural productivity reduces the Foreign industrial labor share. Moreover, higher agricultural productivity in one country raises the other country's industrial labor share.*

Proof. See Lemma 2 and Lemma 3. ■

Intuitively, the ambiguous effect of higher Home agricultural productivity on the Home industrial labor share arises from the simultaneous operation of two competing forces — household consumption pattern and international trade specialization — as discussed in the Introduction. Specifically, it facilitates satisfying the agricultural subsistence requirement (i.e., it reduces η^h/A^h), while at the same time raising the agricultural consumption ratio (i.e., it raises $\left(b^h\right)^*$). Furthermore, the change in the household consumption pattern operates only in Home, and thereby only in Home we have the ambiguous effect of higher domestic agricultural productivity on domestic industrial labor share. This ambiguous result in Home explains the competing perspectives discussed in the Introduction: higher agricultural productivity fostering industrial development versus higher agricultural productivity hindering industrial development. The former downplays the role of international trade, whereas the latter privileges it, leading to the opposite conclusion. In Section 4.4, we shed further light on this aspect of our analysis by examining two special cases that capture the essence of these competing perspectives.

4 Agricultural productivity and industrial takeoff

In this section, we discuss how agricultural productivity affects the transition of the economy from pre-industrial stagnation to modern innovation-driven growth. Additionally, we demonstrate the significant role of international agricultural trade in shaping this process.

4.1 The pre-industrial era

In the pre-industrial era, spending on innovation yields negative profits as firm size is not sufficiently large. Hence, in country j, we have $n_t^j = z_t^j = 0$. Furthermore, the industrial labor share is constant by Lemma 2 (i.e., $\dot{l}_{Y,t}^j/l_{Y,t}^j = 0$). Therefore, according to (29), the growth rate of output per capita is $g_t^j = \sigma^j n_t^j + z_t^j + \dot{l}_{Y,t}^j/l_{Y,t}^j = 0$. Using the law of motion of the state variable given in (30) yields

$$\frac{\dot{x}_t^j}{x_t^j} = \lambda - (1-\sigma^j)n_t^j = \lambda, \tag{34}$$

which shows that firm size grows exponentially at the constant rate λ and crosses the finite threshold x_N^j for variety-expanding innovation at the finite time $T_N^j = \frac{1}{\lambda}\log\left(x_N^j/x_0^j\right)$ given initial condition x_0^j. Note that despite the common growth rate of the two populations, the takeoff time is country-specific because it also depends on the threshold x_N^j.

4.2 The first phase of the industrial era

In the first phase of the industrial era, variety-expanding innovation occurs, but quality-improving innovation does not. Specifically, in country j we have $n_t^j > 0$ but $z_t^j = 0$. Also, the industrial labor share is constant by Lemma 2. Therefore, according to (29), the growth rate of output per capita is $g_t^j = \sigma^j n_t^j$. Using the Euler equation (3), the rate of return to entry (26) and the fact that $\dot{c}_t^j / c_t^j = \dot{y}_t^j / y_t^j = g_t^j$, we obtain

$$n_t^j = n_1(x_t^j) = \frac{1}{\beta^j} \left[\mu^j - 1 - \frac{\phi^j}{x_t^j \left(l_Y^j\right)^*} \right] + \lambda - \rho^j, \tag{35}$$

where $n_1(x_t^j)$ denotes the growth rate of variety in the first phase of the industrial era. $n_1(x_t^j)$ is positive if and only if

$$x_t^j > x_N^j \equiv \frac{\phi^j}{\left[\mu^j - 1 - \beta^j(\rho^j - \lambda)\right] \left(l_Y^j\right)^*}, \tag{36}$$

which shows that the threshold x_N^j for variety-expanding innovation is decreasing in the domestic industrial labor share $(l_Y^j)^*$. According to Proposition 1, we have dx_N^h/dA^hambiguous, $dx_N^h/dA^f < 0$, $dx_N^f/dA^h < 0$ and $dx_N^f/dA^f > 0$. In words, the effect of higher Home agricultural productivity is ambiguous: while it reduces the Foreign threshold for variety-expanding innovation, it may either increase or decrease the Home threshold for variety-expanding innovation due to its ambiguous effect on the Home industrial labor share. In addition, higher Foreign agricultural productivity increases the Foreign threshold for variety-expanding innovation and decreases the Home threshold for variety-expanding innovation.

Using (30) and (35) yields

$$\frac{\dot{x}_t^j}{x_t^j} = \frac{1 - \sigma^j}{\beta^j} \left\{ \frac{\phi^j}{x_t^j \left(l_Y^j\right)^*} - \left[\mu^j - 1 - \beta^j \left(\frac{\sigma^j \lambda}{1 - \sigma^j} + \rho^j \right) \right] \right\}. \tag{37}$$

Under condition (31), x_t^j grows throughout the first phase of the industrial era and later on crosses the threshold x_Z^j for quality-improving innovation. Using $g_t^j = \sigma^j n_t^j$ and (35) yields the transitional growth rate

$$g_t^j = \frac{\sigma^j}{\beta^j} \left[\mu^j - 1 - \frac{\phi^j}{x_t^j \left(l_Y^j\right)^*} \right] - \sigma^j(\rho^j - \lambda), \tag{38}$$

which shows that a larger industrial labor share causes a higher transitional growth rate. According to Proposition 1, which characterizes how agricultural productivity affects the industrial labor shares, we obtain the corresponding effects of agricultural productivity on transitional growth rate. We summarize these results in Proposition 2.

Proposition 2 *(Effects of agricultural productivity in the first phase of the industrial era) Higher Home agricultural productivity has an ambiguous effect on the Home takeoff and post-takeoff transitional growth rate, whereas higher Foreign agricultural productivity delays the Foreign takeoff and lowers the Foreign post-takeoff transitional growth rate. Furthermore, higher agricultural productivity in one country hastens the other country's takeoff and raises the other country's post-takeoff transitional growth rate.*

Proof. Proved in the text. ∎

4.3 The second phase of the industrial era

As x_t^j grows over time and eventually crosses the threshold x_Z^j for quality-improving innovation, the economy enters the second phase of the industrial era. In this phase, the growth rate of industrial output per capita is $g_t^j = \sigma_t^j n_t^j + z_t^j$ since the industrial labor share remains constant. We use the Euler equation (3), the rates of return to quality improvement (23) and variety expansion (26), and the fact that $\dot{c}_t^j / c_t^j = \dot{y}_t^j / y_t^j = g_t^j$ to derive the transitional growth rate

$$g_t^j = \alpha^j [(\mu^j - 1) x_t^j \left(l_Y^j\right)^* - \phi^j] - \rho^j, \tag{39}$$

which shows that a larger industrial labor share leads to a higher transitional growth rate. The two components of this growth rate are the innovation rates:

$$n_t^j = n_2(x_t^j) = \frac{(1-\alpha^j)(\mu^j - 1) - \beta^j(\rho^j - \lambda) - [(1-\alpha^j)\phi^j - \rho^j] \frac{1}{x_t^j \left(l_Y^j\right)^*}}{\beta^j - \frac{\sigma^j}{x_t^j \left(l_Y^j\right)^*}}, \tag{40}$$

$$z_t^j = z_2(x_t^j) = \frac{\beta^j \left\{ \left[\alpha^j - \frac{\sigma^j}{\beta^j x_t^j \left(l_Y^j\right)^*} \right] [(\mu^j - 1) x_t^j \left(l_Y^j\right)^* - \phi^j] - [(1-\sigma^j)\rho^j + \sigma^j \lambda] \right\}}{\beta^j - \frac{\sigma^j}{x_t^j \left(l_Y^j\right)^*}}, \tag{41}$$

where $n_2(x_t^j)$ and $z_2(x_t^j)$ are, respectively, the growth rate of variety and quality in the second phase of the industrial era. Equation (41) says that $z_2(x_t^j) > 0$ if and only if

$$x_t^j > x_Z^j \equiv \underset{x_t^j}{\arg\text{solve}} \left\{ \frac{(1-\sigma^j)\rho^j + \sigma^j \lambda}{(\mu^j - 1) x_t^j \left(l_Y^j\right)^* - \phi^j} = \alpha^j - \frac{\sigma^j}{\beta^j x_t^j \left(l_Y^j\right)^*} \right\}, \tag{42}$$

which shows that the threshold x_Z^j for quality-improving innovation is decreasing in the domestic industrial labor share $\left(l_Y^j\right)^*$.

According to Proposition 1, we have dx_Z^h / dA^h ambiguous, $dx_Z^h / dA^f < 0$, $dx_Z^f / dA^h < 0$ and $dx_Z^f / dA^f > 0$. In words, an improvement in the Home agricultural productivity has an ambiguous effect on the Home threshold for quality-improving innovation while it lowers the Foreign threshold for quality-improving innovation. On the other hand, an improvement in the Foreign agricultural productivity lowers the Home threshold for quality-improving innovation and raises the Foreign threshold for quality-improving innovation.

We substitute (40) into (30) to obtain the dynamics of x_t^j in country j as

$$\frac{\dot{x}_t^j}{x_t^j} = \frac{\frac{(1-\alpha^j)\phi^j - \left(\rho^j + \frac{\sigma^j}{1-\sigma^j}\lambda\right)}{x_t^j \left(l_Y^j\right)^*} - \left[(1-\alpha^j)(\mu^j - 1) - \beta^j \left(\rho^j + \frac{\sigma^j}{1-\sigma^j}\lambda\right)\right]}{\left[\beta^j - \frac{\sigma^j}{x_t^j \left(l_Y^j\right)^*}\right] \frac{1}{1-\sigma^j}}, \tag{43}$$

which implies that under condition (31), x_t^j grows over time and eventually converges to

$$(x^j)^* = \frac{(1-\alpha^j)\phi^j - \left(\rho^j + \frac{\sigma^j \lambda}{1-\sigma^j}\right)}{(1-\alpha^j)(\mu^j - 1) - \beta^j \left(\rho^j + \frac{\sigma^j \lambda}{1-\sigma^j}\right)} \frac{1}{\left(l_Y^j\right)^*}. \tag{44}$$

The growth rate of industrial output per capita is

$$(g^j)^* = \alpha^j \left[(\mu^j - 1) \frac{(1-\alpha^j)\phi^j - \left(\rho^j + \frac{\sigma^j \lambda}{1-\sigma^j}\right)}{(1-\alpha^j)(\mu^j - 1) - \beta^j \left(\rho^j + \frac{\sigma^j \lambda}{1-\sigma^j}\right)} - \phi^j \right] - \rho^j. \tag{45}$$

This growth rate is independent of the industrial labor share and thus of agricultural productivity—indeed of any factor determining the scale of economic activity. The intuition for this scale invariance property follows from (44), which says that in the steady state, firm size $x_t^j l_{Y,t}^j = (x^j)^* (l_Y^j)^*$ is independent of any parameter related to scale. Consequently, agricultural productivity, has no effect on firm size and thereby has no effect on the steady-state growth rate. We summarize the effects of agricultural productivity in the second phase of the industrial era in Proposition 3.

Proposition 3 *(Effects of agricultural productivity in the second phase of the industrial era) Higher Home agricultural productivity has an ambiguous effect on the Home threshold for quality-improving innovation and on the Home transitional growth rate, whereas higher Foreign agricultural productivity raises the Foreign threshold for quality-improving innovation and lowers the Foreign transitional growth rate. Furthermore, higher agricultural productivity in one country lowers the other country's threshold for quality-improving innovation and increases the other country's transitional growth rate. In both Home and Foreign, the steady-state growth rate is unaffected by agricultural productivity in either country.*

Proof. Proved in the text. ■

4.4 Two special cases

To illuminate further the role of agricultural trade, we now consider two special cases, which isolate two competing forces. First, we shut down the Home subsistence requirement for agricultural goods (i.e., $\eta^h = 0$). Second, we shut down agricultural international trade between two countries. In other words, Home has no preference for the imported agricultural good (i.e., $\delta^h = 1$).

4.4.1 No subsistence requirement ($\eta^h = 0$)

To examine the role of this modification, we need only to track the equations in Home. When we shut down the Home subsistence requirement for agricultural goods ($\eta^h = 0$), the term $1 - \eta^h / A^h$ in (32) becomes 1 and the term $b_t^h = \left(q_t^h - \eta^h\right) / m_t^h$ becomes $b_t^h = q_t^h / m_t^h$. The key change is the first one, because it makes the industrial labor share monotonic in the agricultural consumption ratio $\left(b^h\right)^*$. Specifically, we have

$$(l_Y^h)^* = \begin{cases} \dfrac{(1+\psi^h)\delta^h \left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h+\gamma^h)\left[\delta^h \left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]} & 0 \le x_t^h \le x_N^h \\ \dfrac{(1+\psi^h)\delta^h \left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{\left[1+\psi^h + \frac{1-\theta^h + \frac{\beta^h \theta^h}{\mu^h}(\rho^h - \lambda)}{1-\theta^h}\gamma^h\right]\delta^h \left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)} & x_N^h < x_t^h < \infty \end{cases}. \tag{46}$$

Lemma 3 changes accordingly but still says that the agricultural consumption ratio is increasing in the Home agricultural productivity. It then follows that all of the effects that we study become unambiguous. In particular, under $\eta^h = 0$, the Home agricultural productivity unambiguously decreases the Home industrial labor share and thereby decreases its transitional growth rate. Moreover, whereas in the general case dx_N^h/dA^h and dx_Z^h/dA^h are ambiguous, under $\eta^h = 0$ they become unambiguously positive, that is, $dx_N^h/dA^h > 0$ and $dx_Z^h/dA^h > 0$. In words, higher Home agricultural productivity raises the thresholds for variety-expanding innovation and quality-improving innovation, respectively.

Thus, when the Home subsistence requirement is eliminated, the effect of the Home agricultural productivity is no longer ambiguous. The intuition is that the Home preferences become homothetic, and the model therefore no longer features a channel for the changing consumption pattern of the Home representative household. Consequently, specialization through international trade becomes the only force governing labor reallocation in each country. We summarize these results in Proposition 4.

Proposition 4 *(Effects of agricultural productivity without subsistence requirement) Without subsistence requirement, the effects of agricultural productivity are as in the general case with the only change that higher Home agricultural productivity unambiguously delays the Home takeoff and lowers its transitional growth rate.*

Proof. Proved in the text. ■

4.4.2 No agricultural trade ($\delta^h = 1$)

Without agricultural trade, the Home representative household consumes only its domestic agricultural good and does not import agricultural good. The key implication of the absence of agricultural trade is that the Home expenditure share on its own agricultural good no longer depends on the relative prices of agricultural goods; see (5) and (7). The Home industrial labor share becomes

$$(l_Y^h)^* = \begin{cases} \frac{1+\psi^h}{1+\psi^h+\gamma^h}\left(1-\frac{\eta^h}{A^h}\right) & 0 \le x_t^h \le x_N^h \\ \frac{1+\psi^h}{1+\psi^h+\left[1+\frac{\theta^h}{1-\theta^h}\frac{\beta^h}{\mu^h}(\rho^h-\lambda)\right]\gamma^h}\left(1-\frac{\eta^h}{A^h}\right) & x_N^h < x_t^h < \infty \end{cases}. \tag{47}$$

All effects operating through agricultural prices have washed out. Consequently, the Home industrial labor share is unambiguously increasing in its own agricultural productivity A^h, and is independent of the Foreign agricultural productivity A^f. Moreover, the Foreign industrial labor share becomes

$$(l_Y^f)^* = \begin{cases} \frac{1+\psi^f}{1+\psi^f+\gamma^f} & 0 \le x_t^f \le x_N^f \\ \frac{1+\psi^f}{1+\psi^f+\left[1+\frac{\theta^f}{1-\theta^f}\frac{\beta^f}{\mu^f}(\rho^f-\lambda)\right]\gamma^f} & x_N^f < x_t^f < \infty \end{cases}, \tag{48}$$

which is independent of both its own agricultural productivity A^f, and the Home agricultural productivity A^h. The reason is that Foreign has no subsistence requirement, which is the only remaining channel in (47) through which agricultural productivity could have an effect.

The equations describing the thresholds for variety-expanding and quality-improving innovation are the same as in the general case, see (36) and (42). Therefore, an improvement in the Home agricultural productivity reduces the Home thresholds for variety-expanding and quality-improving innovation, hastening the takeoff and raising the transitional growth rate. In addition, unlike in the general case, now the improvement in the Home agricultural productivity has no effect on Foreign. Furthermore, the Foreign agricultural productivity has no effect on either Home or Foreign. The general intuition behind this special case is that it eliminates the force of specialization through international trade, leaving only the changing consumption pattern of the Home household to drive the reallocation of labor within its own country. We summarize the results of this section in Proposition 5.

Proposition 5 *(Effects of agricultural productivity without agricultural trade) An improvement in the Home agricultural productivity hastens the Home takeoff and raises the Home transitional growth rate, while it has no effect on Foreign. Furthermore, an improvement in the Foreign agricultural productivity has no effect on either Home or Foreign.*

Proof. Proved in the text. ■

5 Quantitative analysis

In this subsection, we calibrate the general model to data to perform a quantitative analysis. Given that the analytical results on the effects of the Home agricultural productivity on its economy are ambiguous, we conduct numerical experiments to examine these effects and see how they differ across countries. We designate the US as Foreign and designate, respectively, China and Japan, as Home in two separate numerical experiments. This setup is reasonable because China and Japan are among the largest importers of US agricultural products while they export a very small amount of agricultural products to the US, aligning with the assumption of our theoretical model.[28] Furthermore, the US is the leading exporter of major agricultural goods in the world. For Japan, the US is the largest supplier of agricultural goods. For China, the US was often the largest supplier of agricultural goods before early 2010s; even after the US-China trade war, the US has consistently ranked among China's top suppliers. Therefore, it is natural to treat the US as the main trading partner and neglect the rest of the world in our two-country framework, for tractability.

The model features the following parameters $\{\lambda, \epsilon, \theta^h, \theta^f, \alpha^h, \alpha^f, \sigma^h, \sigma^f, \mu^h, \mu^f, \rho^h, \rho^f, \psi^h, \psi^f, \phi^h, \phi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, L_0^h/L_0^f, \eta^h/A^h, A^h/A^f\}$. We set the average population growth rate to $\lambda = 0.01$. We follow Iacopetta and Peretto (2021) to set the subjective discount rate to a conventional value of 0.03. The labor share of output is set to 0.67 in the US and Japan, and 0.55 in China.[29] We set the markup ratio to 1.3 for China, 1.4 for Japan, and 1.5 for

[28] China, Mexico, Canada and Japan are the top four importers of the US agricultural goods. However, Canada and Mexico do not fit our model as they are not only major importers but also major exporters in agriculture to the US.

[29] See Song *et al.* (2011), Backus *et al.* (2017) and Grossman and Oberfield (2021).

the US.[30] According to Feenstra *et al.* (2018) and Bajzik *et al.* (2020), we set the elasticity of substitution between domestic and foreign agricultural goods ϵ to 3. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers to $1-\alpha^h = 1-\alpha^f = 0.833$ and the social return of variety to $\sigma^h = \sigma^f = 0.25$. Next, we calibrate the remaining parameters $\{\psi^h, \psi^f, \phi^h, \phi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, L_0^h/L_0^f, \eta^h/A^h, A^h/A^f\}$ by matching moments in the US and China or Japan. After that, we conduct simulations to explore how agricultural productivity affects economic growth.

5.1 China and the US

We first designate China as Home and the United States as Foreign. We calibrate L_0^h/L_0^f to 4.49 by using the ratio of the average population in China and the US, and A_{aver}^h/A_{aver}^f to 0.043 by using the ratio of average agricultural value added per agricultural worker between the two countries. Furthermore, we calibrate the parameters $\{\psi^h, \psi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, \eta^h/A_{aver}^h\}$ by matching the following moments: 1.2% for the agricultural consumption share of GDP in the US,[31] 46.7% for the consumption share of GDP in China, 62.0% for the consumption share of GDP in the US, 9.8% for the share of agricultural imports in total agricultural consumption in China, 11.2% for the import share of GDP in China, 10.5% for the import share of GDP in the US, 0.34 for the income elasticity of the agricultural expenditure share in China,[32] and 97.4% for the non-agricultural labor share in the US. Finally, we use the long-run TFP growth rates, which are 0.90% in China and 0.58% in the US, to calibrate the remaining parameters $\{\phi^h, \phi^f\}$.[33]

Intuitively, moments for the import share of GDP and the share of agricultural imports in total agricultural consumption are informative about the preference parameters for imported industrial goods (i.e., ψ^h and ψ^f). When industrial trade becomes more extensive, the balanced-trade condition induces a corresponding adjustment in agricultural trade, and these joint adjustments are reflected in the import share of GDP. In addition, the agricultural consumption share of GDP and the income elasticity of the agricultural expenditure share are informative about the preference parameters for agricultural goods (i.e., γ^h and γ^f). A higher preference weight on agricultural goods means that households place more value on agricultural consumption or have a stronger taste for agricultural goods. As a result, households reduce their food expenditure more gradually as their income grows. Moreover, the share of agricultural imports in total agricultural consumption is the most informative moment for identifying the reliance on agricultural imports (i.e., $1-\delta^h$), because stronger reliance directly raises the volume of agricultural imports. The reliance on agricultural imports is also closely related to moments for the import share of GDP and the industrial labor share, since a large volume of agricultural imports affects both the overall import share and the allocation of labor across sectors. Finally, the income elasticity of the agricultural expenditure share is most informative about the labor

[30] See empirical estimates of the markup ratios in Fan *et al.* (2018), De Loecke *et al.* (2020), Lu and Yu (2015) and Morrison (1992).

[31] Data source: Food and Agriculture Organization Data.

[32] See Nakamura *et al.* (2016) and Alder *et al.* (2022).

[33] Data source: Federal Reserve Bank of St. Louis.

requirement associated with the subsistence constraint (i.e., η^h/A^h_{aver}).[34] The intuition is that a heavy subsistence requirement forces households to devote most of their income to food at low income levels, so the agricultural expenditure share drops rapidly as income rises, generating a large income elasticity of the agricultural expenditure share. Table 3 summarizes the parameter values.

Table 3: Calibrated parameters (China and the US)

θ^h	α^h	σ^h	μ^h	ρ^h	ψ^h	ϕ^h	γ^h	β^h	λ	δ^h	η^h/A^h_{aver}
0.45	0.167	0.25	1.3	0.03	0.240	0.167	0.087	5.125	0.01	0.82	0.038
θ^f	α^f	σ^f	μ^f	ρ^f	ψ^f	ϕ^f	γ^f	β^f	ϵ	L^h_0/L^f_0	A^h_{aver}/A^f_{aver}
0.33	0.167	0.25	1.5	0.03	0.172	0.059	0.020	11.638	3.00	4.49	0.043

Figure 1 plots the historical series of agricultural value added per agricultural worker in China, which increases from $579 in 1978 to $5,380 in 2019.[35] Agricultural productivity in China rises steadily before the 1990s and then accelerates significantly from the early 1990s onward. Figure 2 plots the historical series of agricultural value added per agricultural worker in the US, which increases from $16,470 in 1978 to $69,946 in 2019. As agricultural productivity changes over time in both China and the US, we calibrate the time-varying ratios A^h/A^f and η^h/A^h from 1978 to 2019 (see Figure 3 and Figure 4).[36] These changes can be considered as unanticipated permanent shocks in our model. Then, we simulate the path of the technology growth rate driven by simultaneous changes in agricultural productivity in both China and the US. As a counterfactual comparison, we also simulate another technology growth path, for which agricultural productivity A^h in China remains at its initial value in 1978.

[34] Since the subsistence requirement is not directly observable in the data, we conduct sensitivity checks using alternative values of the income elasticity in Appendix E. Specifically, we consider 0.22 from Boppart (2014), which implies a lower subsistence requirement, and 0.49 from Alder *et al.* (2022), which implies a higher subsistence requirement. The former downplays the consumption-pattern force, whereas the latter amplifies it. In all cases, the quantitative effects of agricultural productivity remain non-negligible.

[35] In the quantitative analysis, we measure agricultural productivity in China, Japan and the US using agricultural value added per agricultural worker, in constant 2015 US dollars.

[36] Here we use $(\eta^h/A^h_{aver})(A^h_{aver}/A^h)$ to calibrate the time path of η^h/A^h.

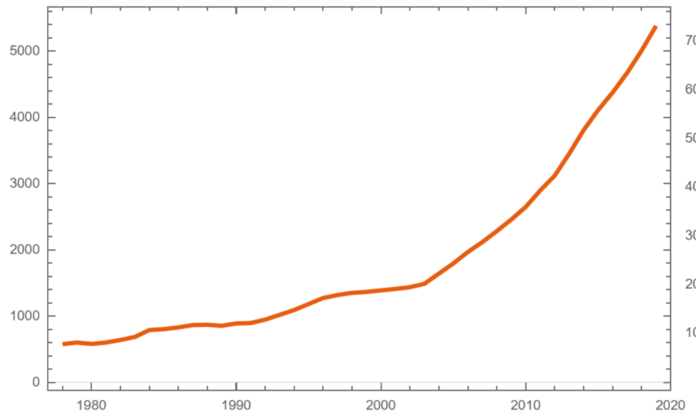

Figure 1: Agricultural productivity in China.

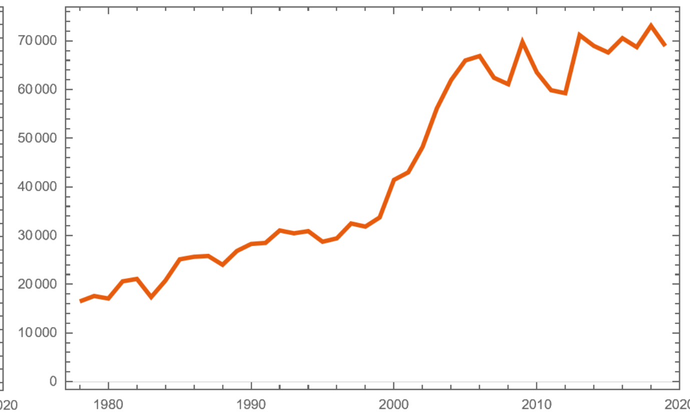

Figure 2: Agricultural productivity in the US.

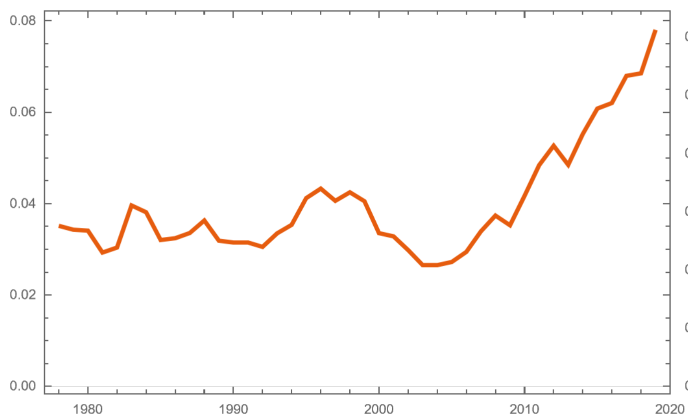

Figure 3: Calibrated path of A^h/A^f.

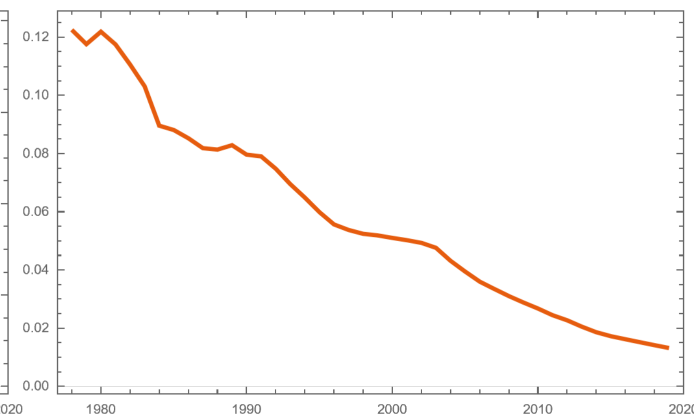

Figure 4: Calibrated path of η^h/A^h.

Figure 5 shows the simulated paths of the technology growth rate in China. With improvement in China's agricultural productivity, the simulated growth rate in China gradually rises from 1978 to early 1990s. Then, from the early 1990s onward, there is an acceleration in the rise of China's technology growth, as agricultural productivity in China experiences a rapid surge. This rapid rise in agricultural productivity since the early 1990s triggers China's earlier entry into the era of quality-driven growth, leading to a higher growth rate thereafter. The simulated growth rate rises from 0.24% in 1978 to 0.73% in 2019. Without improvement in agricultural productivity, China would not enter the era of quality-driven growth until the early 2000s. Furthermore, in the absence of agricultural productivity improvement in China, the simulated growth rate rises from 0.24% to only 0.45% in 2019. Comparing these two cases, agricultural productivity improvement in China is responsible for an additional increase in the growth rate of 0.28%. In the data, the TFP growth rate in China increased from 0.24% in 1978-1999 to 1.19% in 2000-2019; therefore, our model with agricultural productivity improvement in China accounts for about 30 percent of the increase in China's TFP growth in this period.

Figure 6 shows the simulated paths of the technology growth rate in the US. With the improvement in agricultural productivity in China, the simulated technology growth rate in the US slightly rises from 0.55% in 1978 to 0.60% in 2019. Without the improvement in agricultural productivity in China, the simulated technology growth rate in the US decreases from 0.55% in 1978 to 0.46% in 2019. In the data, the TFP growth rate in US increases from 0.53% to 0.62% during the same period. Therefore, the simulated US technology growth rate, incorporating improvements in China's agricultural productivity, aligns more closely with the data. This also suggests that improvement in agricultural productivity in China contributes positively to economic growth in the US, which is also consistent with our theoretical prediction.[37]

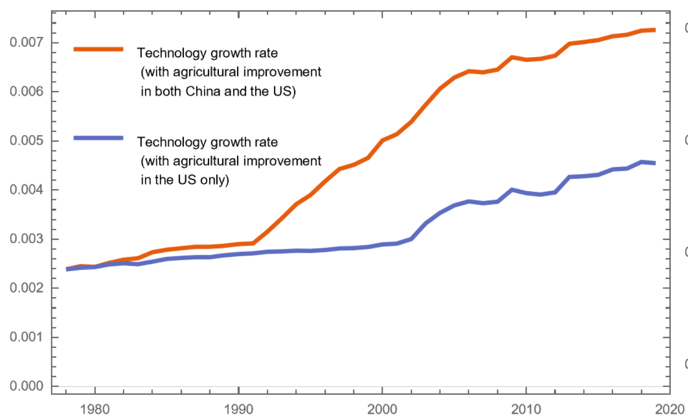

Figure 5: Simulated growth rate in China.

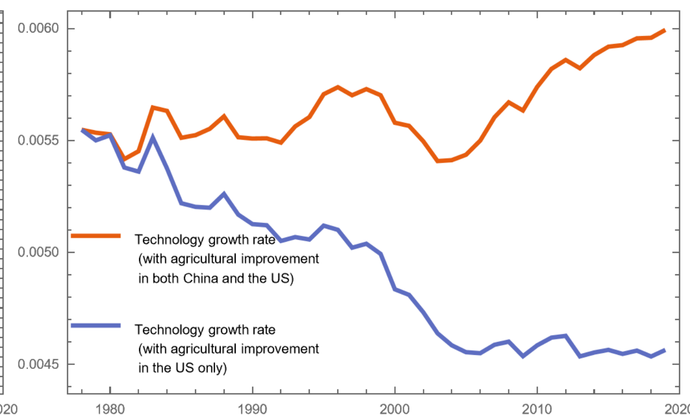

Figure 6: Simulated growth rate in the US.

5.2 Japan and the US

We now designate Japan as Home and the US as Foreign. Given the absence of an estimate for the income elasticity of the agricultural expenditure share in Japan, we use the same calibrated value of η^h as in the China-US case. More importantly, this allows us to examine how agricultural productivity in China and Japan accounts for differences in their economies under varying degrees of reliance on agricultural trade. We calibrate L_0^h/L_0^f to 0.45 by using the ratio of the average population in Japan and the US, and A_{aver}^h/A_{aver}^f to 0.39 by using the ratio of average agricultural value added per agricultural worker between two countries. We calibrate the parameters $\{\psi^h, \psi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h\}$ by matching the following moments: 1.2% for the agricultural consumption share of GDP in the US, 52.0% for the consumption share of GDP in Japan, 62.0% for the consumption share of GDP in the US, 30.3% for the share of agricultural imports in total agricultural consumption in Japan, 17.5% for the import share of GDP in Japan, 10.5% for the import share of GDP in the US, and 97.4% for the non-agricultural labor share in the US. Finally, we use the long-run TFP growth rates, which are 0.37% in Japan and 0.58% in the US, to calibrate the remaining parameters $\{\phi^h, \phi^f\}$.[38] Table 4 summarizes the

[37] In Appendix E, we do another counterfactual exercise to shut down the agricultural improvement in the US. We find that the US agricultural productivity growth leads to a higher transitional growth rate in China; see Figure E1.

[38] Data source: Federal Reserve Bank of St. Louis.

parameter values.

Table 4: Calibrated parameters (Japan and the US)

θ^h	α^h	σ^h	μ^h	ρ^h	ψ^h	ϕ^h	γ^h	β^h	λ	δ^h	η^h/A^h_{1978}
0.33	0.167	0.25	1.4	0.03	0.326	0.194	0.055	6.113	0.01	0.74	0.004
θ^f	α^f	σ^f	μ^f	ρ^f	ψ^f	ϕ^f	γ^f	β^f	ϵ	L^h_0/L^f_0	A^h_{aver}/A^f_{aver}
0.33	0.167	0.25	1.5	0.03	0.172	0.059	0.020	11.638	3.00	0.45	0.39

In Figure 7, we plot the path of agricultural productivity in Japan, which increases from \$10,328 in 1978 to \$18,142 in 2019. The path of agricultural productivity in the US is given in Figure 2. We calibrate the time-varying ratios A^h/A^f and η^h/A^h from 1978 to 2019 (see Figure 8 and Figure 9) and then treat them as a sequence of unanticipated permanent shocks to simulate the technology growth path. We also conduct a counterfactual exercise in which there is no improvement in agricultural productivity in Japan by holding A^h constant at its initial level in 1978.

Figure 10 shows the simulated paths of the technology growth rate in Japan. In contrast to the Chinese case, improvement in domestic agricultural productivity exhibits negative effects on the simulated growth rate in Japan. The simulated technology growth rate in Japan decreases as its domestic agricultural productivity increases over time. With the improvement in Japan's agricultural productivity, the simulated growth rate slightly declines from 0.62% in 1978 to 0.58% in 2019. Conversely, without the improvement in Japan's agricultural productivity, the simulated growth rate increases from 0.62% in 1978 to 0.61% in 2019. Comparing these two cases, agricultural productivity improvement in Japan is responsible for an additional decrease in the growth rate of 0.03%. The average TFP growth rate in Japan was 0.64% in 1978-1999, and it declined to 0.46% in 2000-2019. Therefore, our model with agricultural productivity improvement in Japan accounts for about 15 percent of the decline in TFP growth in Japan.

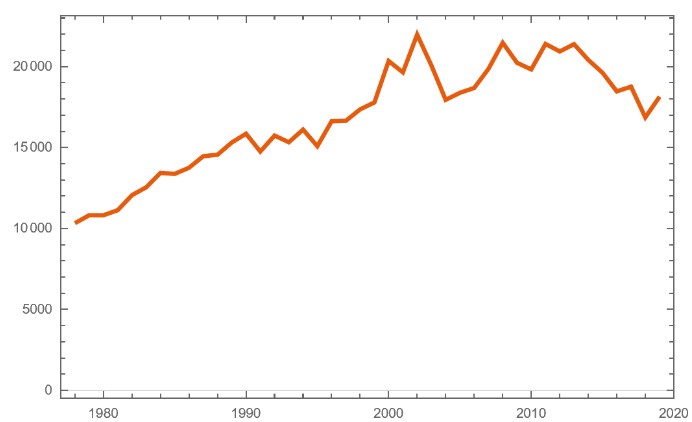

Figure 7: Agricultural productivity in Japan.

Figure 11 plots the simulated paths of the technology growth rate in the US. The improvement in Japan's agricultural productivity results in a decrease in the simulated growth rate in the US, from 0.55% in 1978 to 0.52% in 2019. In the absence of improvement in Japan's agricultural productivity, the simulated growth rate in the US declines from 0.55% in 1978 to 0.50% in 2019. As mentioned before, in the data, the TFP growth rate in the US increased from 0.53% to 0.62% during this period. Therefore, the simulated US growth rate, when accounting for the improvement in Japan's agricultural productivity, aligns more closely with the data. This also suggests that improvement in agricultural productivity in Japan contributes positively to economic growth in the US.[39]

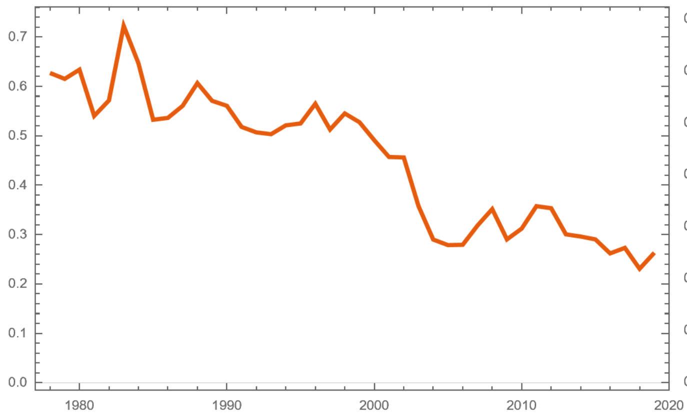

Figure 8: Calibrated path of A^h/A^f.

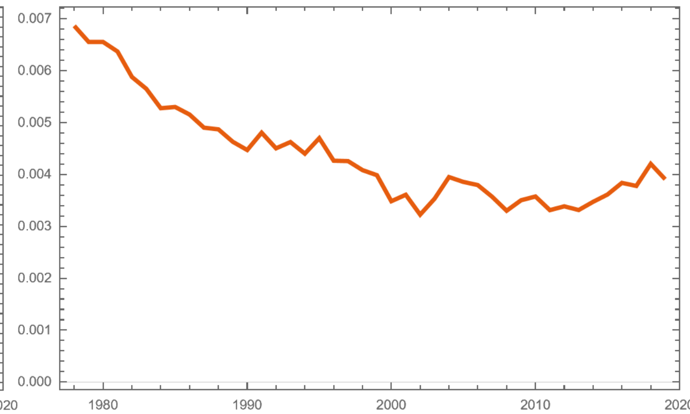

Figure 9: Calibrated path of η^h/A^h.

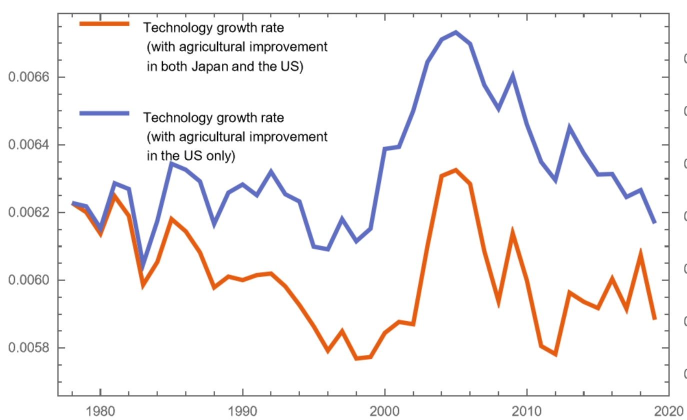

Figure 10: Simulated growth rate in Japan.

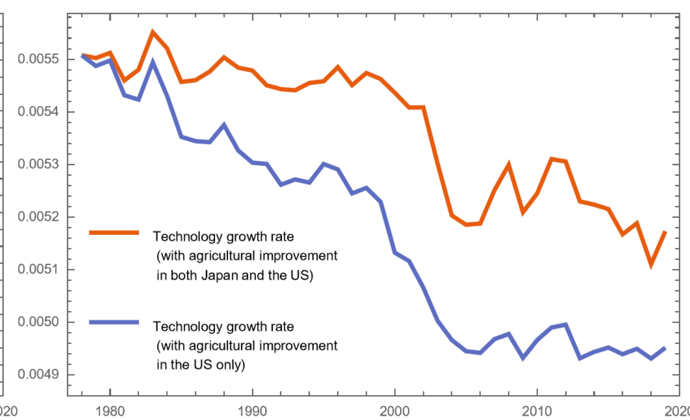

Figure 11: Simulated growth rate in the US.

5.3 Intuition for the different effects in China and Japan

In the previous sections, we have calibrated our model to data in the US and its two major agricultural importers: China and Japan. Then, we have performed quantitative analyses to

[39] In Appendix E, we do another counterfactual experiment to shut down the agricultural improvement in the US, and find that the US agricultural productivity growth contributes to Japan's technological growth; see Figure E2.

explore the overall growth effects of agricultural productivity in China/Japan on economic growth in China/Japan and the US. Our counterfactual exercises reveal that improvements in agricultural productivity in both Japan and China positively affect US growth, aligning with our theoretical predictions.

Moreover, we find that agricultural productivity improvement in China and Japan can explain about 30 percent and 15 percent, respectively, of the changes in domestic TFP growth. However, the quantitative effects of agricultural productivity on domestic technology growth differ between China and Japan. Specifically, in China, improving agricultural productivity increases its domestic technology growth, whereas this effect is negative in Japan.[40] These drastically different implications can be explained as follows.

First of all, it is useful to note that the calibrated values of δ^h for China and Japan are 0.82 and 0.74, respectively. Suppose we have the absence of agricultural trade (i.e., $\delta^h = 1$). Then, an increase in domestic agricultural productivity does not affect import substitution because household only consumes domestic agricultural good. In this case, the presence of subsistence agricultural requirement implies that an improvement in agricultural productivity releases labor from the agricultural sector to the industrial sector, leading to a positive effect on technology growth. This overall positive effect also applies to China given its high calibrated value of $\delta^h = 0.82$. In other words, the specialization force through trade is relatively weak in China, and the consumption pattern force dominates.

In the case of Japan, its calibrated value of $\delta^h = 0.74$ is relatively low. In this case, the higher level of agricultural imports in Japan implies that it has a stronger incentive for import substitution due to the substitutability between domestic and imported agricultural products. In other words, the specialization force through trade is relatively strong in Japan. Therefore, an increase in domestic agricultural productivity gives rise to agricultural import substitution and leads to a reallocation of labor from the industrial sector to the agricultural sector, leading to a negative effect on technology growth. Our quantitative results illustrate that the degree of reliance on agricultural imports influences the effects of agricultural productivity on economic growth.

5.4 Implications of trade liberalization

The quantitative results show that agricultural trade plays an important role in shaping the effects of agricultural productivity on industrial technological progress. In this subsection, we further discuss the role of trade in both agricultural and industrial sectors.

In our model, a larger value of $1 - \delta^h$ (i.e., a smaller δ^h) represents trade liberalization in the agricultural sector. This implies that the Home representative household consumes more imported agricultural goods. Since domestic and imported agricultural goods are substitutes, Home consumes less of domestic agricultural good, which reduces the size of its agricultural sector and lowers its agricultural labor requirement, thereby raising the Home industrial labor share. At the same time, the increased demand for the Foreign agricultural good pulls more labor into the Foreign agricultural sector, thereby expanding the size of its agricultural sector and reducing its industrial labor share. Therefore, agricultural trade liberalization hastens industrialization in Home while stifling industrialization in Foreign.

[40] These contrasting quantitative results are consistent with our empirical estimates for China and Japan in Section 2.

Furthermore, a larger ψ^h represents industrial trade liberalization, which implies that Home has more industrial imports. As a result, the increased demand for the Foreign industrial good raises the Foreign industrial labor share and hastens industrialization in Foreign. However, for Home, industrial trade liberalization has two competing effects on its industrialization. To satisfy the balanced-trade condition, more industrial imports may cause (1) Home to export more industrial good, and (2) Home to reduce its agricultural imports. The first effect raises the Home industrial labor share and hastens industrialization, while the second effect reduces the Home industrial labor share and stifles industrialization. To be more specific, in the absence of agricultural trade, industrial trade liberalization contributes to industrialization in Home. However, with agricultural trade, the overall effect on the Home industrialization becomes ambiguous.

6 Extensions

In this section, we consider two extensions to our baseline model. Section 6.1 introduces land as a factor input for agricultural production. Section 6.2 introduces a spillover effect from industry to agriculture.

6.1 Arable land

In our baseline model, labor is the only factor input in the agricultural sector. In this extension, we follow Bustos *et al.* (2016) and consider a more realistic setting with fixed arable land. Specifically, we modify agricultural production function as

$$Q_t^j = A^j \left(\Lambda^j\right)^{\varphi} \left(L_{A,t}^j\right)^{1-\varphi} \left(\bar{L}_{A,t}^j\right)^{\varphi}, \tag{49}$$

where $\varphi \in [0, 1)$ nests our baseline model as a special case when $\varphi = 0$. Λ denotes a fixed supply of arable land. $\bar{L}_{A,t}^j$ captures a positive spillover effect, under which more agricultural labor may discover new agricultural technologies to make agricultural production more productive, and maintains constant returns to scale with respect to agricultural labor at the aggregate level. In equilibrium, we have $L_{A,t}^j = \bar{L}_{A,t}^j$. Profit maximization yields the price of the agricultural good

$$p_{A,t}^j = \frac{1}{(1-\varphi)\left(\Lambda^j\right)^{\varphi}} \frac{w_t^j}{A^j}, \tag{50}$$

and the payment for land

$$\omega_t^j \Lambda^j = \frac{\varphi}{1-\varphi} w_t^j L_{A,t}^j, \tag{51}$$

where ω_t^j denotes the rental price of land in country j.

In addition to asset income and wage income, the representative household also earns income from land. Therefore, the asset-accumulation equation (2) becomes

$$\dot{a}_t^j = (r_t^j - \lambda) a_t^j + w_t^j \left(l_{A,t}^j + l_{Y,t}^j\right) + \frac{\omega_t^j \Lambda^j}{L_t^j} - p_{Y,t}^j c_t^j - p_{Y,t}^{-j} \iota_t^j - p_{A,t}^j q_t^j - p_{A,t}^{-j} m_t^j. \tag{52}$$

With these modifications, the dynamic system of our model does not change substantially, and expressions for consumption-output ratios remain identical to those in Lemma 1. Furthermore, Lemma 3 still holds, implying that agricultural consumption ratio is increasing in the Home agricultural productivity but decreasing in the Foreign agricultural productivity. Combining (5), (11), (14), (49), (50) and $l^j_{A,t} + l^j_{Y,t} = 1$ yields the steady-state industrial labor share in Home and Foreign, respectively:

$$l^h_{Y,t} = (l^h_Y)^* = \left[1 + \frac{(1-\varphi)\gamma^h}{1-\theta^h} \frac{\delta^h \left((b^h)^*\right)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h \left((b^h)^*\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h} \left(\frac{c^h}{y^h}\right)^*\right]^{-1} \left[1 - \frac{\eta^h}{A^h(\Lambda^h)^\varphi}\right], \tag{53}$$

$$l^f_{Y,t} = (l^f_Y)^* = \left\{1 + \frac{1-\varphi}{1-\theta^f} \left[\gamma^f + \frac{(1-\delta^h)\gamma^h\psi^f}{\psi^h\delta^h \left((b^h)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h+\gamma^h)(1-\delta^h)}\right] \left(\frac{c^f}{y^f}\right)^*\right\}^{-1}. \tag{54}$$

Recall that the Home consumption-output ratio $(c^h/y^h)^*$ is increasing in agricultural consumption ratio $(b^h)^*$, while the Foreign consumption-output ratio $(c^f/y^f)^*$ is decreasing in agricultural consumption ratio $(b^h)^*$. Thus, similar to our baseline model, the Home industrial labor share is decreasing in agricultural consumption ratio, and the Foreign industrial labor share is increasing in agricultural consumption ratio. As a result, the effects of agricultural productivity on industrial labor shares remain qualitatively the same, and the implications for industrial takeoff and economic growth remain the same as in our baseline model.

6.2 Spillovers from the industrial sector

In this subsection, we introduce a spillover effect from the industrial sector to the agricultural sector. The agricultural production function is modified as

$$Q^j_t = A^j (x^j_t)^{\chi^j} L^j_{A,t} = A^j (x^j_t)^{\chi^j} l^j_{A,t} L^j_t, \tag{55}$$

which is increasing in x^j_t, the state-variable component of the quality-adjusted firm size. The parameter $\chi^j \in [0, 1)$ captures the strength of industrial spillovers and nests our baseline model as a special case when $\chi^j = 0$. The price of the domestic agricultural good then becomes

$$p^j_{A,t} = \frac{w^j_t}{A^j (x^j_t)^{\chi^j}}. \tag{56}$$

In this extension, the dynamic system of our model consists of not only dynamics of the consumption-output ratios and the agricultural consumption ratio $\{c^h_t/y^h_t, c^f_t/y^f_t, b^h_t\}$, but also of dynamics of the industrial firm-size state variables $\{x^h_t, x^f_t\}$. The extended model therefore features a higher-dimensional dynamic system in which the evolution of x^j_t varies over time, since our model captures different phases of development in both countries. As a result, fully characterizing the entire transitional dynamics becomes more complicated. However, by restricting our attention to the pre-industrial era, we can still cleanly characterize the key mechanism of our model: agricultural productivity affects industrialization primarily through its impact on industrial labor shares. For this reason, in what follows, we focus on the case in which both Home and Foreign are in the pre-industrial era, where the analysis remains tractable and closely aligned with the baseline model.

The expressions for consumption-output ratios c_t^h/y_t^h and c_t^f/y_t^f are, respectively:

$$\frac{c_t^h}{y_t^h} = \frac{1-\theta^h}{1+\psi^h + \frac{(1-\delta^h)\gamma^h}{\delta\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}, \tag{57}$$

and

$$\frac{c_t^f}{y_t^f} = \frac{1-\theta^f}{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} \tag{58}$$

which are identical to the expressions in Lemma 1. Therefore, a larger agricultural consumption ratio leads to a higher Home consumption-output ratio and a lower Foreign consumption-output ratio. Substituting (5), (11), (55), (56) and $l_{A,t}^j + l_{Y,t}^j = 1$ into (14) yields the industrial labor share in Home and Foreign, respectively:

$$l_{Y,t}^h = \left[1+\frac{\gamma^h}{1-\theta^h}\frac{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}\frac{c_t^h}{y_t^h}\right]^{-1}\left[1-\frac{\eta^h}{A^h(x_t^h)^\chi}\right], \tag{59}$$

and

$$l_{Y,t}^f = \left\{1+\frac{1}{1-\theta^f}\left[\gamma^f + \frac{(1-\delta^h)\gamma^h\psi^f}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\right]\frac{c_t^f}{y_t^f}\right\}^{-1}. \tag{60}$$

Similar to our baseline model, the Home industrial labor share is decreasing in the agricultural consumption ratio, and the Foreign industrial labor share is decreasing in the agricultural consumption ratio.

In this extension, although the agricultural consumption ratio b_t^h changes over time in the pre-industrial era, its comparative statics remain the same as in our baseline model: agricultural consumption ratio b_t^h is increasing in the Home agricultural productivity A^h and decreasing in the Foreign agricultural productivity A^f.[41] Therefore, our Proposition 1 still holds in this extension when Home and Foreign are in the pre-industrial era: higher Home agricultural productivity has an ambiguous effect on the Home industrial labor share, whereas higher Foreign agricultural productivity reduces the Foreign industrial labor share; moreover, higher agricultural productivity in one country raises the other country's industrial labor share.

7 Conclusion

In this study, we developed a two-country open-economy Schumpeterian growth model with an agricultural sector to explore the role of agricultural productivity in the endogenous takeoff of the economy and the subsequent path of economic growth. We find that agricultural trade plays an important role in shaping the effects of agricultural productivity on innovation-driven takeoff and economic growth. Our theoretical results can be summarized as follows.

With agricultural trade and a subsistence requirement, higher domestic agricultural productivity has ambiguous effects on the economy's takeoff time and its transitional growth rate.

[41] See Appendix C for detailed derivations.

Under a greater-than-unity elasticity of substitution between domestic and imported agricultural goods as the data suggests, the specialization force works in the opposite direction of the consumption pattern force governed by the subsistence requirement, which tends to release labor from agricultural production. The tension between these two forces explains the ambiguous result in the general case. With no subsistence requirement, the consumption pattern force no longer operates and the ambiguity goes away: the specialization force implies that higher domestic agricultural productivity increases the demand for domestic agricultural goods, which in turn raises the demand for agricultural labor, thereby delaying the economy's industrialization and lowering its transitional growth rate. In the absence of agricultural trade, the specialization force does not operate and the subsistence requirement on agricultural consumption implies that an improvement in domestic agricultural productivity reallocates labor from agriculture to industry, hastening the economy's takeoff and raising the transitional growth rate.

We investigated this mechanism empirically and quantitatively. In a cross-country panel-data exercise, we find that agricultural productivity has a direct positive relationship with economic growth but the overall relationship becomes weaker and can even become negative when reliance on agricultural imports is sufficiently high. In quantitative counterfactual exercises with the calibrated model, we find that improvement in domestic agricultural productivity accounts for the rise and fall of TFP growth in China and Japan, respectively, and also contributes to the rise of TFP growth in their main trading partner, the US.

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A Empirical results

Table A1: Summary statistics

Variables	Observations	Mean	Sd	Min	Max
Log real GDP per capita	788	8.379	1.438	5.240	11.588
Log non-agricultural real GDP per capita	775	8.247	1.559	4.847	11.586
Log TFP index	540	-0.038	0.188	-1.097	0.781
Log cereal yields per hectare	788	10.063	0.718	7.458	11.910
Cereal import dependency ratio	788	0.350	0.326	0.000	1.000
Log capital stock	752	12.410	2.098	6.497	18.294
Government expenditure share of GDP	758	0.186	0.075	0.007	0.616
Capital depreciation rate	752	0.044	0.012	0.013	0.100
Real interest rate	608	0.112	0.071	0.010	0.624

Data source: Food and Agricultural Organization Data for cereal yields per hectare and cereal import dependency. U.S. Department of Agriculture for agricultural TFP. World Bank Data for real GDP per capita and non-agricultural real GDP per capita. Penn World Table for other variables.

Table A2: Agricultural productivity, trade and transitional growth (Sub-sample)

	log GDP per capita		log non-agri GDP per capita		log TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{jt}	0.492***	0.258**	0.481***	0.254**	0.338***	0.374***
	(0.114)	(0.108)	(0.131)	(0.117)	(0.108)	(0.123)
$A_{jt} \times trade_{jt}$	-0.566***	-0.568***	-0.688***	-0.689***	-0.681***	-0.702***
	(0.157)	(0.180)	(0.180)	(0.196)	(0.153)	(0.148)
$trade_{jt}$	5.388***	5.416***	6.450***	6.538***	6.395***	6.602***
	(1.507)	(1.742)	(1.720)	(1.890)	(1.505)	(1.477)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Period fixed effects	✓	✓	✓	✓	✓	✓
Observations	382	277	371	271	237	237
R^2	0.9803	0.9849	0.9821	0.9865	0.6927	0.7170

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses, clustered at the country level.*

To isolate the effects of agricultural improvements from the potential persistence of the dependent variable, we include the lag of the dependent variable in our empirical specification:

$$y_{jk} = \kappa_1 A_{jk} + \kappa_2 A_{jk} \times trade_{jk} + \kappa_3 trade_{jk} + \kappa_4 y_{j,k-1} + \Gamma\Phi_{jk} + \zeta_j + \zeta_k + \varepsilon_{jk},$$

where k denotes the year. In this estimate, we use annual data instead because the 5-year-average data distorts the dynamic structure and weakens the internal GMM instruments, which may bias the estimate. $y_{j,k-1}$ is the lagged dependent variable, which is measured by log level of real GDP per capita, or the log level of non-agricultural real GDP per capita in year $k-1$.[42] Table A3 reports the results of the dynamic panel regression.

Table A3: Dynamic panel regression

	log GDP per capita		log non-agri GDP per capita	
A_{jt}	0.179***	0.159***	0.142***	0.085*
	(0.034)	(0.042)	(0.041)	(0.044)
$A_{jt} \times trade_{jt}$	-0.169***	-0.171***	-0.115***	-0.097**
	(0.036)	(0.044)	(0.043)	(0.042)
$trade_{jt}$	1.768***	1.745***	1.063**	0.979**
	(0.390)	(0.456)	(0.450)	(0.436)
Lag of dependent variable	✓	✓	✓	✓
Control variables		✓		✓
Country fixed effects	✓	✓	✓	✓
Year fixed effects	✓	✓	✓	✓
Observations	3790	2817	3668	2734
Hansen J test	0.321	0.155	0.298	0.212
$AR(2)$ serial correlation test	0.883	0.214	0.817	0.949

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Windmeijer-corrected robust standard errors are reported in parentheses.*

[42] We do not use TFP as the dependent variable in this dynamic panel regression because annual TFP data is very noisy.

Table A4: Agricultural productivity, trade, and growth (aggregate-level proxies)

	log GDP per capita		log non-agri GDP per capita		log TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{jt}	0.274***	0.283***	0.282***	0.264***	0.205**	0.213**
	(0.095)	(0.097)	(0.105)	(0.101)	(0.091)	(0.089)
$A_{jt} \times trade_{jt}$	-0.081***	-0.107***	-0.099***	-0.123***	-0.067***	-0.087***
	(0.015)	(0.017)	(0.016)	(0.019)	(0.011)	(0.014)
$trade_{jt}$	0.058***	0.069***	0.069***	0.079***	0.018*	0.038**
	(0.017)	(0.018)	(0.016)	(0.020)	(0.009)	(0.016)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Period fixed effects	✓	✓	✓	✓	✓	✓
Observations	667	564	659	558	506	506
R^2	0.9903	0.9923	0.9901	0.9924	0.6734	0.7096

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses, clustered at the country level.*

Table A5: Geo-climatic condition and cereal yields

	log of cereal yields
soil quality index × log of annual precipitation	0.225***
	(0.062)
Country fixed effects	✓
Year fixed effects	✓
F-statistic	35.551
Observations	3785
R^2	0.9321

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses, clustered at the country level.*

Table A6: Relationship between agricultural productivity, trade and economic growth (IV)

	log GDP per capita		log non-agri GDP per capita		log TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
$\hat{A}_{jt}$	1.464***	0.746***	1.688***	0.806***	0.449***	0.558***
	(0.126)	(0.215)	(0.132)	(0.227)	(0.087)	(0.176)
$\hat{A}_{jt} \times trade_{jt}$	-0.258*	-0.272**	-0.320**	-0.374**	-0.363***	-0.401***
	(0.147)	(0.136)	(0.154)	(0.145)	(0.127)	(0.133)
$trade_{jt}$	2.538*	2.709**	3.102**	3.729**	3.467***	3.876***
	(1.454)	(1.337)	(1.521)	(1.434)	(1.246)	(1.329)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Observations	701	561	688	554	498	4981
R^2	0.9873	0.9908	0.9876	0.9911	0.6541	0.6817

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses, clustered at the country level. Because precipitation exhibits global trends over time, including period fixed effects would absorb much of the variation in our geo-climate-based cereal yields. Therefore, period fixed effects are excluded from the IV regression to preserve sufficient variation in the instrument.*

Table A7: Agricultural productivity, trade and economic growth (Bartik-style measure for agricultural reliance)

	log GDP per capita		log non-agri GDP per capita		log TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
$\hat{A}_{jt}$	1.532***	0.692***	1.748***	0.832***	0.370***	0.524***
	(0.142)	(0.184)	(0.151)	(0.191)	(0.089)	(0.168)
$\hat{A}_{jt} \times I\left(trade_{j,1991} \times \frac{Price_{j,t-1}}{Price_{j,t}}\right)$	-0.857***	-0.838**	-0.945***	-1.124***	-0.562**	-0.739**
	(0.302)	(0.372)	(0.322)	(0.398)	(0.276)	(0.295)
$I\left(trade_{j,1991} \times \frac{Price_{j,t-1}}{Price_{j,t}}\right)$	8.734***	8.799**	9.349***	11.584***	5.205*	7.064**
	(3.285)	(3.888)	(3.519)	(4.183)	(2.910)	(3.015)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Observations	651	523	642	519	475	475
R^2	0.9890	0.9926	0.9889	0.9927	0.6538	0.6894

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses, clustered at the country level. Because precipitation exhibits global trends over time, including period fixed effects would absorb much of the variation in our geo-climate-based cereal yields. Therefore, period fixed effects are excluded from the IV regression to preserve sufficient variation in the instrument.*

B Intratemporal equilibrium

Substituting (10) into (7) yields

$$b_t^h = \left(\frac{\delta^h}{1-\delta^h} \frac{w_t^f}{w_t^h} \frac{A^h}{A^f} \right)^{\epsilon} = \left(\frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{p_{Y,t}^f y_t^f}{p_{Y,t}^h y_t^h} \frac{l_{Y,t}^h}{l_{Y,t}^f} \frac{A^h}{A^f} \right)^{\epsilon}, \tag{B.1}$$

where the second equality uses (14). We rearrange (B.1) as

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{p_{Y,t}^h c_t^h}{p_{Y,t}^f c_t^f} \frac{c_t^f / y_t^f}{c_t^h / y_t^h} \frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{A^h}{A^f}. \tag{B.2}$$

We use (4), (6) and (27) to obtain

$$\frac{p_{Y,t}^h c_t^h}{p_{Y,t}^f c_t^f} = \frac{\psi^f}{\psi^h + \frac{(1-\delta^h)\gamma^h}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}} \frac{L_t^f}{L_t^h} = \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{L_t^f}{L_t^h}. \tag{B.3}$$

Then, we substitute (B.3) into (B.2) to obtain

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{L_t^f}{L_t^h} \frac{c_t^f / y_t^f}{c_t^h / y_t^h} \frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{A^h}{A^f}, \tag{B.4}$$

which holds at any time t. For brevity, we define the left-hand side of (B.4) as the following function:

$$F(\cdot) \equiv \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{L_t^f}{L_t^h} \frac{c_t^f / y_t^f}{c_t^h / y_t^h} \frac{l_{Y,t}^f}{l_{Y,t}^h}. \tag{B.5}$$

We substitute (5), (9), (10), (11) and $l_{A,t}^h + l_{Y,t}^h = 1$ into (14) to derive the Home industrial labor share as

$$l_{Y,t}^h = \left[1 + \frac{\gamma^h}{1-\theta^h} \frac{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h} \frac{c_t^h}{y_t^h}\right]^{-1} \left(1 - \frac{\eta^h}{A^h}\right). \tag{B.6}$$

We substitute (6), (8), (9), (10), (11), (B.3) and $l_{A,t}^f + l_{Y,t}^f = 1$ into (14) to derive the Foreign industrial labor share

$$l_{Y,t}^f = \left\{1 + \frac{1}{1-\theta^f} \left[\gamma^f + \frac{\gamma^h (1-\delta^h) \psi^f}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right] \frac{c_t^f}{y_t^f}\right\}^{-1}. \tag{B.7}$$

Furthermore, we use (9), (10) and (11) to derive the payments for agricultural labor in Home and Foreign, respectively:

$$w_t^h l_{A,t}^h = p_{A,t}^h q_t^h, \tag{B.8}$$

$$w_t^f l_{A,t}^f = p_{A,t}^f \left(q_t^f + m_t^h \frac{L_t^h}{L_t^f}\right). \tag{B.9}$$

Substituting (B.8), (B.9) and $l^j_{A,t} + l^j_{Y,t} = 1$ into (2), the asset-accumulation equations in Home and Foreign can be expressed as

$$\dot{a}^h_t = (r^h_t - \lambda)a^h_t + w^h_t l^h_{Y,t} - p^h_{Y,t}c^h_t - p^f_{Y,t}\iota^h_t - p^f_{A,t}m^h_t, \tag{B.10}$$

$$\dot{a}^f_t = (r^f_t - \lambda)a^f_t + w^f_t l^f_{Y,t} - p^f_{Y,t}c^f_t - p^h_{Y,t}\iota^f_t + p^f_{A,t}m^h_t \frac{L^h_t}{L^f_t}. \tag{B.11}$$

B.1 Intratemporal equilibrium: both Home and Foreign are in the pre-industrial era

In the pre-industrial era, both $\dot{a}^h_t$ and $\dot{a}^f_t$ are zero due to the absence of asset accumulation during this period. Therefore, (B.10) and (B.11) can be rearranged as

$$p^h_{Y,t}c^h_t = w^h_t l^h_{Y,t} - p^f_{Y,t}\iota^h_t - p^f_{A,t}m^h_t, \tag{B.12}$$

$$p^f_{Y,t}c^f_t = w^f_t l^f_{Y,t} - p^h_{Y,t}\iota^f_t + p^f_{A,t}m^h_t \frac{L^h_t}{L^f_t}. \tag{B.13}$$

Then, we substitute (4), (6) and (14) into (B.12) to obtain the expression for the Home consumption-output ratio

$$\frac{c^h_t}{y^h_t} = \frac{1-\theta^h}{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}, \tag{B.14}$$

which shows that c^h_t/y^h_t is a function of b^h_t. In addition, we substitute (4), (6), (14) and (B.3) into (B.13) to obtain the expression for the Foreign consumption-output ratio

$$\frac{c^f_t}{y^f_t} = \frac{1-\theta^f}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}, \tag{B.15}$$

which shows that c^f_t/y^f_t is also a function of b^h_t. Substituting (B.6), (B.7), (B.14) and (B.15) into (B.5), we re-express $F(\cdot)$ as

$$F(\cdot) = \left(b^h_t\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)} \frac{1}{1-\frac{\eta^h}{A^h}} \frac{L^f_0}{L^h_0} \frac{1-\theta^f}{1-\theta^h} \frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f}, \tag{B.16}$$

in which b^h_t is the only endogenous variable and $F(\cdot)$ is monotonically increasing in b^h_t for any $\epsilon \in (1,\infty)$.[43] Since (B.4) holds at any time t, b^h_t must be equal to a unique value $(b^h)^*$ and remain constant. Consequently, according to (B.14) and (B.15), the consumption-output ratios c^h_t/y^h_t and c^f_t/y^f_t jump to their unique stationary levels.

[43] See Appendix C for a detailed derivation.

B.2 Intratemporal equilibrium: Home is in the pre-industrial era while Foreign is in the industrial era

As Home is still in the pre-industrial era, the expression for the steady-state c_t^h/y_t^h is identical to (B.14). When Foreign enters the industrial era (i.e., $x_t^f > x_N^f$), variety-expanding innovation takes place and the free-entry condition holds. Using $a_t^f = N_t^f V_t^f / L_t^f$, $\theta^f p_{Y,t}^f Y_t^f = \mu^f N_t^f X_t^f$ and the free-entry condition (24) yields

$$a_t^f = \frac{\beta^f \theta^f}{\mu^f} p_{Y,t}^f y_t^f. \tag{B.17}$$

Therefore, we have

$$\frac{\dot{a}_t^f}{a_t^f} = \frac{\dot{y}_t^f}{y_t^f} + \frac{\dot{p}_{Y,t}^f}{p_{Y,t}^f}. \tag{B.18}$$

We use (4), (6), (14), (B.3), (B.11), (B.17) and (B.18) to derive

$$\frac{\dot{y}_t^f}{y_t^f} = (r_t^f - \lambda) + \frac{\mu^f}{\beta^f \theta^f}(1-\theta^f) - \frac{\mu^f}{\beta^f \theta^f}\left[1 + \psi^f - \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]\frac{c_t^f}{y_t^f} - \frac{\dot{p}_{Y,t}^f}{p_{Y,t}^f}. \tag{B.19}$$

We combine (3) and (B.19) to obtain

$$\frac{\dot{c}_t^f}{c_t^f} - \frac{\dot{y}_t^f}{y_t^f} = -(\rho^f - \lambda) - \frac{\mu^f}{\beta^f \theta^f}(1-\theta^f) + \frac{\mu^f}{\beta^f \theta^f}\left[1 + \psi^f - \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]\frac{c_t^f}{y_t^f}. \tag{B.20}$$

To simplify the expressions of the dynamic system, we define $\varsigma_t^h \equiv y_t^h/c_t^h$ and $\varsigma_t^f \equiv y_t^f/c_t^f$. In this notation, we rewrite (B.14) as

$$\varsigma_t^h = \left[1 + \psi^h + \frac{(1-\delta^h)\gamma^h}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}\right]\frac{1}{1-\theta^h}. \tag{B.21}$$

The dynamic system in (B.20) can be re-expressed as

$$\dot{\varsigma}_t^f = \underbrace{\left[(\rho^f - \lambda) + \frac{\mu^f}{\beta^f \theta^f}(1-\theta^f)\right]\varsigma_t^f - \frac{\mu^f}{\beta^f \theta^f}\left[1 + \psi^f - \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]}_{\equiv \varsigma(\varsigma_t^f, b_t^h)}, \tag{B.22}$$

where we define the right-hand side of (B.22) as $\varsigma(\varsigma_t^f, b_t^h)$. We combine (B.4), (B.6) and (B.7) to obtain

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)} \frac{\varsigma_t^h+\frac{\gamma^h}{1-\theta^h}\frac{\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{\varsigma_t^f+\frac{1}{1-\theta^f}\left[\gamma^f+\frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\right]} = \underbrace{\frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}\frac{L_0^h}{L_0^f}\left(1-\frac{\eta^h}{A^h}\right)}_{\equiv\Gamma}. \tag{B.23}$$

For brevity, we define the right-hand side of (B.23) as Γ. Substituting (B.21) into (B.23) yields

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)} \frac{1+\psi^h+\gamma^h}{\varsigma_t^f+\frac{1}{1-\theta^f}\left[\gamma^f+\frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\right]} = \Gamma(1-\theta^h). \tag{B.24}$$

For any $\epsilon \in (1,\infty)$, the left-hand side of (B.24) increases from 0 to ∞ as b_t^h increases from 0 to ∞, while it decreases from a finite value to 0 as ς_t^f increases from 0 to ∞. Therefore, there uniquely exists a function, $\vartheta(\cdot)$, such that $b_t^h \equiv \vartheta(\varsigma_t^f)$, which is increasing in ς_t^f with $\vartheta(0) > 0$ and $\lim_{\varsigma_t^f\to\infty}\vartheta(\varsigma_t^f)\to\infty$.

We solve (B.24) for ς_t^f to obtain

$$\varsigma_t^f = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\frac{1+\psi^h+\gamma^h}{\Gamma} - \left[\frac{\gamma^f}{1-\theta^f}+\frac{1}{1-\theta^f}\frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\right]. \tag{B.25}$$

Then, substituting (B.25) into (B.22) yields

$$\begin{aligned}\dot{\varsigma}_t^f =& \frac{\psi^f\left(1+\psi^h+\gamma^h\right)\left[(\rho^f-\lambda)+\frac{\mu^f}{\beta^f\theta^f}(1-\theta^f)\right]}{\Gamma}\frac{\left(b_t^h\right)^{\frac{1}{\epsilon}}\left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h(b_t^h)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)} \\ &-\frac{\rho^f-\lambda}{1-\theta^f}\frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}-\frac{\gamma^f(\rho^f-\lambda)}{1-\theta^f}-\frac{\mu^f\left(1+\psi^f+\gamma^f\right)}{\beta^f\theta^f},\end{aligned} \tag{B.26}$$

which shows that $\dot{\varsigma}_t^f$ is monotonically increasing in b_t^h. Given that $b_t^h = \vartheta(\varsigma_t^f)$ is increasing in ς_t^f, $\dot{\varsigma}_t^f$ is monotonically increasing in ς_t^f when $\epsilon \in (1,\infty)$.

Given that $\dot{\varsigma}_t^f$ is increasing in ς_t^f for any $\epsilon \in (1,\infty)$, ς_t^f jumps to its unique level and remains constant if $\dot{\varsigma}_t^f < 0$ when $\varsigma_t^f = 0$ and $\dot{\varsigma}_t^f > 0$ when $\varsigma_t^f \to \infty$. According to (B.22), $\dot{\varsigma}_t^f$ is strictly negative when $\varsigma_t^f = 0$, as the following inequality always holds:

$$1+\psi^f-\frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h\vartheta(0)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)} > 0. \tag{B.27}$$

In addition, as $\rho^f > \lambda$, the following inequality

$$(\rho^f-\lambda)+\frac{\mu^f}{\beta^f\theta^f}(1-\theta^f) > 0, \tag{B.28}$$

always holds, which suggests that $\dot{\varsigma}_t^f$ is strictly positive when $\varsigma_t^f \to \infty$. Therefore, ς_t^f jumps to its unique level and remains constant. Since $b_t^h = \vartheta(\varsigma_t^f)$ is an increasing function of ς_t^f, b_t^h must equal a unique value when ς_t^f is constant. Furthermore, ς_t^h jumps to a unique level when b_t^h is constant. Then we complete the proof that the dynamic system is stable. From $c_t^h/y_t^h = 1/\varsigma_t^h$, $c_t^f/y_t^f = 1/\varsigma_t^f$ and $b_t^h = \vartheta(\varsigma_t^f)$, the endogenous variables c_t^h/y_t^h, c_t^f/y_t^f and b_t^h reach their own unique level and remain constant. Specifically, when Home is the pre-industrial era and Foreign is the industrial era,[44] the expression for steady-state c_t^h/y_t^h is identical to (B.14), and the expression for steady-state c_t^f/y_t^f in the industrial era is given by

$$\frac{c_t^f}{y_t^f} = \frac{1 - \theta^f + \frac{\beta^f \theta^f}{\mu^f}(\rho^f - \lambda)}{1 + \psi^f - \frac{\psi^f (1-\delta^h)\gamma^h}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon - 1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}}. \tag{B.29}$$

[44]The proof for the case in which both Home and Foreign are in the industrial era is relegated to an online appendix available upon request.

C Proofs

Dynamic optimization of the Home representative household. The current-value Hamiltonian of the Home representative household is

$$H_t^h = \ln c_t^h + \psi^h \ln \iota_t^h + \gamma^h \ln \left[\delta^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}} + (1-\delta^h)(m_t^h)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} + \xi_t^h \dot{a}_t^h, \tag{C.1}$$

where ξ_t^h is co-state variable on $\dot{a}_t^h$. Substituting (2) into (C.1) yields

$$\frac{\partial H_t^h}{\partial q_t^h} = 0 \Rightarrow p_{A,t}^h (q_t^h - \eta^h) = \frac{\delta^h \gamma^h p_{Y,t}^h c_t^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}} + (1-\delta^h)(m_t^h)^{\frac{\epsilon-1}{\epsilon}}}, \tag{C.2}$$

$$\frac{\partial H_t^h}{\partial m_t^h} = 0 \Rightarrow p_{A,t}^f m_t^h = \frac{(1-\delta^h) \gamma^h p_{Y,t}^h c_t^h (m_t^h)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}} + (1-\delta^h)(m_t^h)^{\frac{\epsilon-1}{\epsilon}}}. \tag{C.3}$$

Then, we rewrite (C.2) as (5) and rewrite (C.3) as (6). In addition, we use (C.2) and (C.3) to obtain (7). ■

Dynamic optimization of monopolistic firms. The current-value Hamiltonian of monopolistic firms is

$$H_t^j(i) = \Pi_t^j(i) - p_{Y,t}^j R_t^j(i) + \varkappa_t^j(i) \dot{Z}_t^j(i) + \zeta_t^j(i) \left[\mu^j p_{Y,t}^j - p_{X,t}^j(i) \right] \tag{C.4}$$

where $\varkappa_t^j(i)$ is co-state variable on $\dot{Z}_t^j$ and $\zeta_t^j(i)$ is the multiplier on the markup price $p_{X,t}^j(i) \leq \mu^j p_{Y,t}^j$. We substitute (13), (16) and (17) into (C.4) and take derivative to derive

$$\frac{\partial H_t^j(i)}{\partial p_{X,t}^j(i)} = \frac{\partial \Pi_t^j(i)}{\partial p_{X,t}^j(i)} - \zeta_t^j(i) = 0, \tag{C.5}$$

$$\frac{\partial H_t^j(i)}{\partial R_t^j(i)} = -p_{Y,t}^j + \varkappa_t^j(i) = 0 \Rightarrow \varkappa_t^j(i) = p_{Y,t}^j, \tag{C.6}$$

$$\frac{\partial H_t^j(i)}{\partial Z_t^j(i)} = \alpha^j \left\{ \left[p_{X,t}^j(i) - p_{Y,t}^j \right] \left[\frac{\theta^j}{p_{X,t}^j(i)/p_{Y,t}^j} \right]^{\frac{1}{1-\theta^j}} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} - p_{Y,t}^j \phi^j \right\} \left[\frac{Z_t^j}{Z_t^j(i)} \right]^{1-\alpha^j} = r_t^j \varkappa_t^j(i) - \dot{\varkappa}_t^j(i). \tag{C.7}$$

If $p_{X,t}^j(i) < \mu^j p_{Y,t}^j$, then we have $\zeta_t^j(i) = 0$ and $p_{X,t}^j(i) = p_{Y,t}^j/\theta^j$. If the constraint on $p_{X,t}^j(i)$ is binding, we have $\zeta_t^j(i) > 0$ and $p_{X,t}^j(i) = \mu^j p_{Y,t}^j$. As we employ the assumption that $\mu^j \in (1, 1/\theta^j)$, the price of intermediate good $X_t^j(i)$ is given by $p_{X,t}^j(i) = \min\{\mu^j p_{Y,t}^j, p_{Y,t}^j/\theta^j\} = \mu^j p_{Y,t}^j$. Substituting (13), (C.6) and $p_{X,t}^j(i) = \mu^j p_{Y,t}^j$ into (C.7) and imposing symmetry yield (20). ■

Industrial labor shares. Appendix B shows that endogenous variables c_t^h/y_t^h, c_t^f/y_t^f and b_t^h jump to their own unique level and remain constant. Also, it provides the expressions for the industrial labor shares in Home and Foreign, as shown in (B.6) and (B.7). These expressions include the consumption-output ratio c_t^j/y_t^j. We further derive more specific expressions of the industrial labor shares across different eras in country j.

Substituting the Home pre-industrial consumption-output ratio (B.14) into (B.6) yields the Home industrial labor share in the pre-industrial era:

$$l^h_{Y,t} = \frac{(1+\psi^h)\delta^h \left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h+\gamma^h)\left[\delta^h \left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]} \left(1 - \frac{\eta^h}{A^h}\right). \tag{C.8}$$

Substituting the Home consumption-output ratio in the industrial era[45] into (B.6) yields the Home industrial production labor share in the industrial era:

$$l^h_{Y,t} = \frac{(1+\psi^h)\delta^h \left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{\left[1+\psi^h + \frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h \left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)} \left(1 - \frac{\eta^h}{A^h}\right). \tag{C.9}$$

In addition, we substitute the Foreign pre-industrial consumption-output ratio (B.15) into (B.7) to derive the Foreign industrial labor share in the pre-industrial era:

$$l^f_{Y,t} = \frac{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\gamma^f}. \tag{C.10}$$

We also substitute the Foreign consumption-output ratio in the industrial era (B.29) into (B.7) to derive the Foreign industrial labor share in industrial era:

$$l^f_{Y,t} = \frac{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f + \frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f + \frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h+\gamma^h)(1-\delta^h)}}. \tag{C.11}$$

■

Comparative statics of b^h_t with respect to A^h and A^f.

According to (B.4) and (B.5), the following equality holds at any time t

$$F(\cdot) \equiv \left(b^h_t\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h \left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h\delta^h\left(b^h_t\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h+\gamma^h)(1-\delta^h)} \frac{L^f_t}{L^h_t}\frac{c^f_t/y^f_t}{c^h_t/y^h_t}\frac{l^f_{Y,t}}{l^h_{Y,t}} = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}. \tag{C.12}$$

The expression of $F(\cdot)$ varies depending on the era in which country j is situated, due to the differentiated stationary values of consumption-output ratio between the pre-industrial era and the industrial era. In the following, we prove that $F(\cdot)$ is determined by b^h_t. Then we discuss the monotonicity of $F(\cdot)$ with respect to b^h_t. There are three cases: (1) both Home and Foreign are in the pre-industrial era; (2) Home is in the pre-industrial era while Foreign is in the industrial era; and (3) both Home and Foreign are in the industrial era.

[45] See Lemma 1 for its expression.

Case 1: Both Home and Foreign are in the pre-industrial era. When Home and Foreign are both in the pre-industrial era, combining c_t^h/y_t^h in (B.14) and c_t^f/y_t^f in (B.15) yields

$$\frac{c_t^f/y_t^f}{c_t^h/y_t^h} = \frac{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}\frac{1-\theta^f}{1-\theta^h}. \tag{C.13}$$

Then we use (C.8) and (C.10) to obtain

$$\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{1}{1-\frac{\eta^h}{A^h}}\frac{\frac{(1+\psi^h+\gamma^h)\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{(1+\psi^h)\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}}{\frac{1+\psi^f+\gamma^f}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}}. \tag{C.14}$$

Substituting (C.13) and (C.14) into (B.5) yields

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}}\frac{\psi^f\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\frac{1}{1-\frac{\eta^h}{A^h}}\frac{L_0^f}{L_0^h}\frac{1-\theta^f}{1-\theta^h}\frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f},$$

which can be rewritten as

$$F(\cdot) = \frac{\psi^f\left(b_t^h\right)^{\frac{1}{\epsilon}}}{\psi^h+\frac{\gamma^h(1-\delta^h)}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}\frac{1}{1-\frac{\eta^h}{A^h}}\frac{L_0^f}{L_0^h}\frac{1-\theta^f}{1-\theta^h}\frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f}, \tag{C.15}$$

which is increasing in b_t^h for $\epsilon \in (1,\infty)$. Therefore, we complete the proof that $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (1,\infty)$ when both Home and Foreign are in the pre-industrial era.

Case 2: Home is in the pre-industrial era and Foreign is in the industrial era. When Home is in the pre-industrial era and Foreign is in the industrial era, combining c_t^h/y_t^h in (B.14) and c_t^f/y_t^f in (B.29) yields

$$\frac{c_t^f/y_t^f}{c_t^h/y_t^h} = \frac{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h}. \tag{C.16}$$

We then use (C.8) and (C.11) to obtain

$$\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{1}{1-\frac{\eta^h}{A^h}}\frac{\frac{(1+\psi^h+\gamma^h)\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{(1+\psi^h)\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}}{\frac{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}}. \tag{C.17}$$

Substituting (C.16) and (C.17) into (B.5) yields

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right](1+\psi^h+\gamma^h)\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h}\frac{1}{1-\frac{\eta^h}{A^h}}}{\left[\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h},$$

which can be rewritten as

$$F(\cdot) = \frac{\psi^f\left(b_t^h\right)^{\frac{1}{\epsilon}}(1+\psi^h+\gamma^h)\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h}\frac{1}{1-\frac{\eta^h}{A^h}}}{\left[\psi^h+\frac{\gamma^h(1-\delta^h)}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}, \tag{C.18}$$

which is increasing in b_t^h for $\epsilon \in (1,\infty)$. Therefore, we complete the proof that $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (1,\infty)$ when Home is in the pre-industrial era and Foreign is in the industrial era.

Case 3: Both Home and Foreign are in the industrial era. When both Home and Foreign are in the industrial era, combining industrial-era consumption-output ratios c_t^h/y_t^h and c_t^f/y_t^f in Lemma 1 yields

$$\frac{c_t^f/y_t^f}{c_t^h/y_t^h} = \frac{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}. \tag{C.19}$$

We then use (C.9) and (C.11) to obtain

$$\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{1}{1-\frac{\eta^h}{A^h}}\frac{\frac{\left[1+\psi^h+\frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h)\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}}{\frac{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}}. \tag{C.20}$$

Substituting (C.19) and (C.20) into (B.5) yields

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}}\frac{\psi^f\left\{\left[1+\psi^h+\frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)\right\}\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}\frac{1}{1-\frac{\eta^h}{A^h}}}{\left[\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h},$$

which can be rewritten as

$$F(\cdot) = \frac{\psi^f\left(b_t^h\right)^{\frac{1}{\epsilon}}\left[1+\psi^h+\gamma^h+\frac{\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\frac{\delta^h}{\delta^h+(1-\delta^h)/\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}}\right]\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}\frac{1}{1-\frac{\eta^h}{A^h}}}{\left[\psi^h+\frac{\gamma^h(1-\delta^h)}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}, \tag{C.21}$$

which is increasing in b_t^h for $\epsilon \in (1,\infty)$. Therefore, we complete the proof that $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (1,\infty)$ when both Home and Foreign are in the industrial era.

The above analysis shows that, regardless of the phase in which Home and Foreign are, $F(\cdot)$ can be considered as a function of b_t^h. Furthermore, $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (1,\infty)$ (i.e., $F(\underset{+}{b_t^h})$). Recall that (C.12) holds at any time t:

$$F(\cdot) = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}.$$

Therefore, a higher level of the agricultural productivity A^h in Home (A^f in Foreign) implies that the value of $F(\cdot)$ is larger (smaller). Consequently, a higher level of agricultural productivity A^h in Home (A^f in Foreign) leads to a larger (smaller) value of b_t^h. ■

Proofs for extension with industrial spillovers. Substituting (14) and (56) into (7) yields

$$b_t^h = \left[\frac{\delta^h}{1-\delta^h}\frac{w_t^f}{w_t^h}\frac{A^h}{A^f}\left(\frac{x_t^h}{x_t^f}\right)^{\chi}\right]^{\epsilon}, \tag{C.22}$$

which can be rewritten as

$$(b_t^h)^{\frac{1}{\epsilon}}\frac{p_{Y,t}^h c_t^h}{p_{Y,t}^f c_t^f}\frac{c_t^f/y_t^f}{c_t^h/y_t^h}\frac{l_{Y,t}^f}{l_{Y,t}^h}\frac{(x_t^f)^{\chi^f}}{(x_t^h)^{\chi^h}} = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}. \tag{C.23}$$

We substitute (B.3) into (C.23) to obtain

$$\frac{\psi^f\left(b_t^h\right)^{\frac{1}{\epsilon}}\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\frac{L_0^f}{L_0^h}\frac{c_t^f/y_t^f}{c_t^h/y_t^h}\frac{l_{Y,t}^f}{l_{Y,t}^h}\frac{(x_t^f)^{\chi^f}}{(x_t^h)^{\chi^h}} = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}, \tag{C.24}$$

where we define the left-hand side as $G(\cdot)$ for brevity. According to the law of motion for x_t^j given in (30), the ratio $x_t^h/x_t^f = x_0^h/x_0^f$ is constant when both Home and Foreign are in the pre-industrial era. Combining (57), (58), (59), (60) and (C.24) yields

$$G(\cdot) = \frac{\psi^f\left(b_t^h\right)^{\frac{1}{\epsilon}}\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\frac{\left(\frac{x_0^f}{x_0^h}\right)^{\chi^h}\left(x_0^f e^{\lambda t}\right)^{\chi^f-\chi^h}}{1-\frac{\eta^h}{A^h\left(x_0^h e^{\lambda t}\right)^{\chi^h}}}\frac{L_0^f}{L_0^h}\frac{1-\theta^f}{1-\theta^h}\frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f}, \tag{C.25}$$

where we use $x_t^j = x_0^j e^{\lambda t}$ in the pre-industrial era. Since $G(\cdot)$ is increasing in b_t^h and (C.24) holds for any time t, it is straightforward to prove that the agricultural consumption ratio b_t^h has an unique value at any time t. ■

D Low elasticity of substitution between domestic and foreign agricultural goods

In this appendix, we consider an alternative assumption in which the elasticity of substitution between domestic and foreign agricultural goods is less than 1 (i.e., $\epsilon \in (0, 1)$). In other words, domestic and imported agricultural goods are complements.

In this case, Lemma 2 suggests that the Home industrial labor share is increasing in the agricultural consumption ratio, whereas the Foreign industrial labor share is decreasing in it. Specifically, we have $l_Y^h(\underset{+}{b_t^h})$ and $l_Y^f(\underset{-}{b_t^h})$, indicating that the specialization force works in the opposite direction compared with our baseline model. Lemma 3 still holds. Therefore, an improvement in the Home agricultural productivity causes the consumption pattern force and the specialization force to work in the same direction in Home. Proposition 1 becomes

Proposition D1 *(Effects of agricultural productivity on industrial labor shares) If the elasticity of substitution between domestic and foreign agricultural goods is less than one, in both Home and Foreign, higher domestic agricultural productivity raises the domestic industrial labor share while reducing the foreign industrial labor share.*

Intuitively, when domestic and imported agricultural goods are complements, the representative Home household tends to consume them jointly, which implies that the demand for each agricultural good is inelastic. As a result, as the Home agricultural productivity rises and the price of its agricultural good falls, the quantity demanded rises less than one for one, releasing some agricultural labor to the industrial sector. In the meanwhile, Home imports more agricultural good from Foreign due to complementarity, which raises the demand for the Foreign agricultural good and therefore reduces the Foreign industrial labor share. Similarly, as the Foreign agricultural productivity rises, its industrial labor share increases because the demand for the Foreign agricultural good rises less than one for one. In addition, the demand for the Home agricultural good increases to maintain the complementary consumption bundle, which reduces the Home industrial labor share.

As in our baseline model, agricultural productivity affects the Home and Foreign economies through changes in their industrial labor shares, and the mechanism linking industrial labor shares to takeoff and growth remains unchanged. According to Proposition D1, we summarize the effects of agricultural productivity on takeoff and the transitional growth rate under $\epsilon \in (0, 1)$ in Proposition D2.

Proposition D2 *(Effects of agricultural productivity on takeoff and the transitional growth rate) If the elasticity of substitution between domestic and foreign agricultural goods is less than one, in both Home and Foreign, higher agricultural productivity in one country hastens its own takeoff and raises its post-takeoff transitional growth rate, but has the opposite effects on the other country's takeoff and post-takeoff transitional growth rate.*[46]

[46] Detailed derivations are available upon request.

E Quantitative results

E.1 Counterfactual experiments (shutting down the US agricultural improvement)

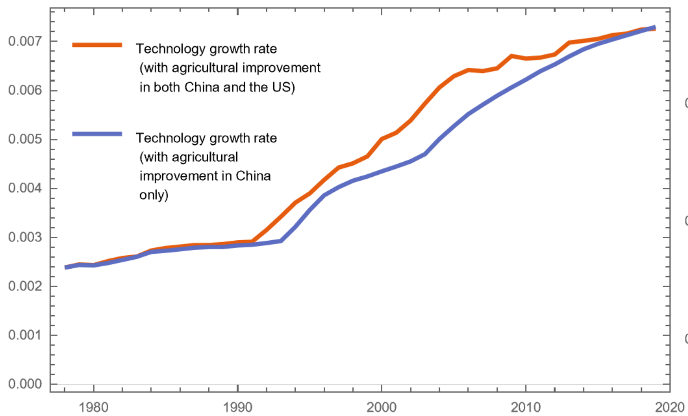

Figure E1: Simulated growth rate in China

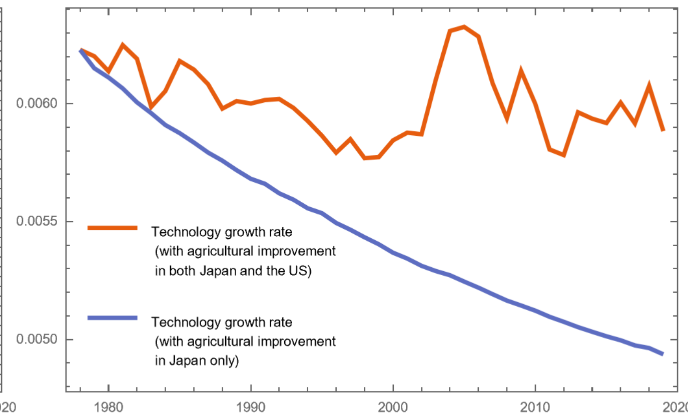

Figure E2: Simulated growth rate in the US

E.2 Sensitivity checks

Since the subsistence requirement is not directly observable in the data, we conduct sensitivity checks using alternative values of the income elasticity of the agricultural expenditure share. Specifically, we first consider a lower income elasticity of 0.22 from Boppart (2014), which yields a lower calibrated value of the subsistence requirement. The quantitative results remain largely unchanged, although domestic agricultural improvement explains a smaller fraction of the increase in China's TFP growth and a larger fraction of the decline in Japan's TFP growth; see Figures E3 and E4. Next, we consider a higher income elasticity of 0.49 from Alder *et al.* (2022), which implies a higher subsistence requirement. In this case, domestic agricultural improvement accounts for a larger fraction of the increase in China's TFP growth and a smaller fraction of the decline in Japan's TFP growth.; see Figures E5 and E6.

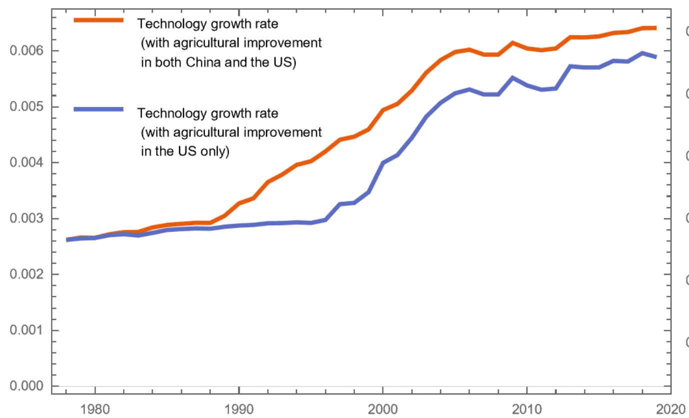

Figure E3: Simulated growth rate in China

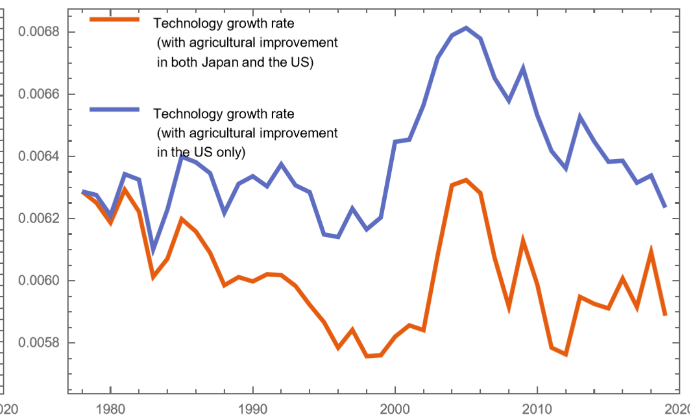

Figure E4: Simulated growth rate in Japan

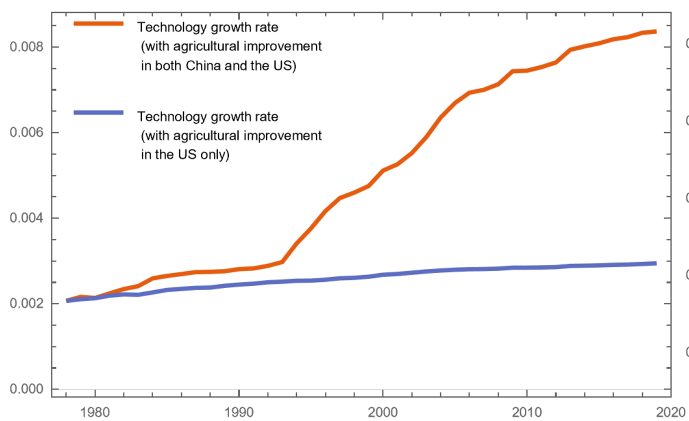

Figure E5: Simulated growth rate in China

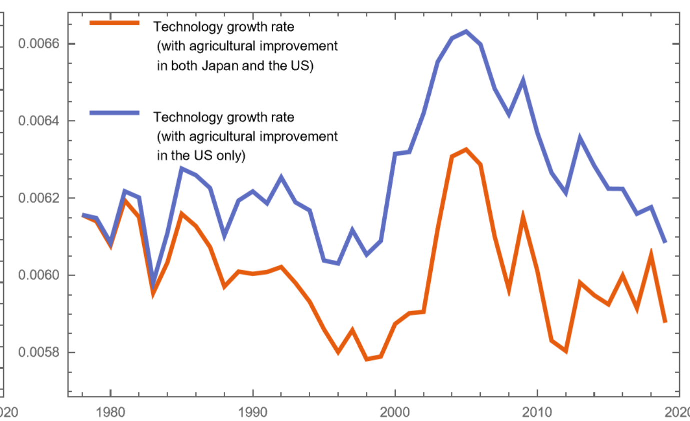

Figure E6: Simulated growth rate in Japan