Early State Formation and Interstate Competition in a Malthusian Economy

Angus C. Chu　　Xilin Wang　　Rongxin Xu

April 2026

Abstract

This study develops a Malthusian growth model with early state formation and interstate competition. It explores how the capacity of early states to collect tax revenue and provide productive public goods affects their population dynamics and interstate competition. We find that there is an optimal tax rate for each state that is increasing in the elasticity of agricultural output with respect to productive public goods. Without interstate competition, there is a threshold value for this elasticity below (equal) which the population converges to a steady state (grows at a constant rate) in the long run. With interstate competition, there is a lower threshold value for the above mentioned elasticity below (equal) which competing states coexist (only one state survives) in the long run. Whether we consider productive public goods as a flow or stock variable, the population-maximizing tax rate for each state is increasing in its elasticity of agricultural output with respect to productive public goods. We use historical data to provide empirical evidence for this theoretical result.

JEL classification: E60, H20, H56, O43
Keywords: Public goods, interstate competition, Malthusian growth theory

Chu: anguscccc@gmail.com. Department of Economics, University of Macau, Macau, China. Wang: xilinwang@fudan.edu.cn. Department of World Economy, School of Economics, Fudan University, Shanghai, China; Research Institute of Innovation and Digital Economy, Fudan University, Shanghai, China. Xu: rongxinecon@gmail.com. School of Economics, Huazhong University of Science and Technology, Wuhan, China. A previous version of this manuscript was circulated under the title "Early State Formation in a Malthusian Economy". The authors would like to thank Areendam Chanda for his helpful comments. The usual disclaimer applies.

> “In the immediate aftermath of the transition to agriculture, most societies maintained the basic tribal frameworks that had prevailed beforehand. [. . .] As tribes did not raise taxes on any meaningful scale, they were generally not engaged in the construction of major public infrastructure, such as irrigation canals, fortifications or temples, [and] the next stage in the political development of agricultural societies was the chiefdom – a hierarchical society consisting of a number of villages or communities governed by a supreme chief. [. . .] Crucially, these hierarchical societies tended to raise taxes or tithes to support the elite and pay for the provision of public infrastructure.” Galor (2022, p. 208)

1 Introduction

Evidence suggests that our species, *Homo sapiens*, emerged about 300,000 years ago. For most of this time, our ancestors were hunter-gatherers. About 12,000 years ago, the Neolithic Revolution occurred, and our ancestors began to undergo the transition from nomadic hunter-gatherers to sedentary farmers. Then, civilizations gradually emerged in agricultural settlements, first in Mesopotamia (present-day Iraq) and then in ancient China, Egypt and the Indus Valley. All these early societies had the needs to raise taxes for the provision of public goods.[1] As Galor (2022, p. 209) writes, “whether benevolent or tyrannical, the inescapable condition for [rulers’] existence was their capacity to raise taxes.” To explore this issue, we develop an agricultural Malthusian growth model with early state formation and interstate competition. Then, we use this growth-theoretic framework to explore how the capacity of early states to collect tax revenue and provide productive public goods, such as irrigation and military defense, affects population dynamics and the expansion or contraction of each state. Our results can be summarized as follows.

In the special case without interstate competition, there is an optimal tax rate in terms of population growth for each state that is increasing in the elasticity of agricultural output with respect to productive public goods. Furthermore, there is a threshold value for this elasticity below which the population converges to a steady state. In this case, the steady-state population size is increasing in the share of tax revenue allocated to public goods and an inverted-U function in the tax rate. If the abovementioned elasticity happens to be equal to the threshold value, then the population size grows at a constant rate in the long run. In this case, the population size of the state keeps growing because expansion in productive public goods and population growth reinforce one another, in a virtuous cycle. In other words, productive public goods can give rise to long-run population growth in the Malthusian epoch. Whether the elasticity is equal to or below the threshold, agricultural output per capita always remains constant in the long run and is increasing in the tax rate.

In the general case with interstate competition, the optimal tax rate for each state is increasing in the elasticity of agricultural output with respect to productive public goods as before. Furthermore, there is a threshold value for this elasticity below which competing states coexist in the long run. In this case, the steady-state population size of each state is increasing in its own share of tax revenue allocated to public goods and an inverted-U function in its own tax rate as before but is now decreasing in the other states’ share of

---

[1] See Svizzero and Tisdell (2014) for a discussion of such public goods in these early states.

tax revenue allocated to public goods and a U-shaped function in their tax rates. If the abovementioned elasticity is equal to its threshold value, then the state with the highest population growth rate eventually becomes a unified empire and converges to a Malthusian steady-state population size, which is increasing in the state's share of tax revenue allocated to public goods and an inverted-U function in its tax rate. The initially positive effect of taxation on interstate competition is due to the importance of productive public goods on agricultural productivity.

Borcan *et al.* (2018) argue that "the earliest states developed the fiscal capacity and coordination needed to achieve increases in productivity, but ultimately limited that productivity due to overcentralization." If we consider that fiscal capacity needs to accumulate over time, then a young state can only levy a low tax rate. As this state accumulates more fiscal capacity, it can levy a higher tax rate and improve agricultural productivity.[2] Eventually, the tax rate becomes too high, perhaps due to political elites' self-interest, and ends up stifling productivity. To explore this story, we also consider an extension of our model with productive public infrastructure as a state variable and find that agricultural productivity initially rises over time but eventually becomes an inverted-U function of the tax rate as in our baseline model. Furthermore, we use historical data to provide empirical evidence for the presence of a population-maximizing tax rate in each state that is increasing in the elasticity of agricultural output with respect to productive public goods.

This study relates most closely to the literature on Malthusian growth theory, which originates from the insight of Malthus (1798). Ehrlich and Lui (1997) provide an excellent review of the early literature. Brander and Taylor (1998) and de la Croix and Dottori (2008) introduce renewable natural resources to the Malthusian growth model to explore the collapse of Easter Island. An influential study by Galor and Moav (2002) introduces natural selection to the Malthusian growth model to explore how it affects the transition from stagnation to sustained growth; see also the subsequent studies in this influential literature by Collins *et al.* (2014), Dalgaard and Strulik (2015), Galor and Michalopoulos (2012), Galor and Ozak (2016), Lagerlof (2007, 2014) and Le Fur and Wasmer (2025).[3] An interesting study by Lagerlof (2021) considers productive and extractive capacities of political elite also in a Malthusian growth model and shows that they lead to multiple equilibria with different extraction rates, population density and output.[4] The present study relates most closely to Chu *et al.* (2024), who explore the Malthusian transition from political fragmentation to a unified empire in an agricultural economy.[5] We generalize the analysis in Chu *et al.* (2024) by introducing productive public goods to their Malthusian growth model with interstate competition.[6]

Therefore, this study also relates to the literature on economic growth and productive public goods. The seminal study by Barro (1990) introduces productive public goods to the

---

[2] See also Bockstette *et al.* (2002) and Chanda and Putterman (2007) for evidence on the advantages of experienced states with early starts.

[3] See Galor (2005, 2011) and Ashraf and Galor (2018) for a comprehensive review of this literature.

[4] Mayshar, Moav and Pascali (2022) provide empirical evidence on why political hierarchy emerged in early states.

[5] Chu (2025) presents a Malthusian growth-theoretic framework to explore the entirety of human evolution, from natural selection of human species to the Neolithic Revolution and the Industrial Revolution.

[6] See Chu (2010) for an analysis of tax and defense competition between states in an AK growth model.

Neoclassical growth model to generate endogenous economic growth in the long run. Subsequent studies by Barro and Sala-i-Martin (1992), Chatterjee and Turnovsky (2012), Futagami *et al.* (1993, 2008), Futagami and Mino (1995), Glomm and Ravikumar (1994), Maebayashi *et al.* (2017) and Turnovsky (1996, 2000) provide different specifications for modelling productive public goods in endogenous growth models.[7] The present study complements these important contributions by modelling productive public goods in the Malthusian growth model and exploring their effects on population dynamics and interstate competition in an agricultural economy.

The rest of this study is organized as follows. Section 2 presents the Malthusian growth model. Section 3 explores population dynamics of each state. Section 4 considers an extension of the model with public investment in productive infrastructure. Section 5 provides empirical evidence. The final section concludes.

2 A Malthusian model with productive public goods

We consider a canonical Malthusian growth model based on Ashraf and Galor (2011). Chu *et al.* (2024) extend their model to consider rent-seeking taxation by political elites and interstate competition between states $i \in \{1, ..., m\}$, where $m \geq 2$ is the initial number of states. Here, we introduce productive public goods to their Malthusian growth model of interstate competition in order to endogenize the level of agricultural productivity.

2.1 Agricultural production

In each state $i \in \{1, ..., m\}$, there are N_t^i adult citizens at time t. Each citizen devotes l_t^i units of labor to farming and receives y_t^i units of agricultural output given by

$$y_t^i = \theta_t^i (l_t^i)^\alpha (z_t^i)^{1-\alpha} = (G_t^i)^\kappa (l_t^i)^\alpha \left(\frac{Z_t^i}{N_t^i} \right)^{1-\alpha}, \tag{1}$$

where $\alpha \in (0, 1)$ is a parameter that determines the intensity α of farming labor l_t^i and the intensity $1-\alpha$ of farming land z_t^i in agricultural production. The amount of land distributed to each citizen in state i at time t is $z_t^i = Z_t^i / N_t^i$, in which Z_t^i is the amount of land occupied by state i. The novel element here is the endogenous level of agricultural productivity $\theta_t^i = (G_t^i)^\kappa$, in which G_t^i is productive public goods in state i and the parameter $\kappa \in (0, 1)$ determines the elasticity of agricultural output with respect to productive public goods.

2.2 Endogenous fertility of citizens

Our model features overlapping generations of citizens. Each citizen lives for two periods: first period as a child, and second period as an adult. The utility function u_t^i of an adult

---

[7] See Glomm and Ravikumar (1997) for a review of this literature.

citizen in state i at time t is given by

$$u_t^i = \beta^i \ln(1 - l_t^i) + (1 - \gamma^i) \ln c_t^i + \gamma^i \ln n_t^i, \tag{2}$$

where $\beta^i \geq 0$ is a parameter that determines the citizen's preference for leisure $1 - l_t^i$, and the parameter $\gamma^i \in (0, 1)$ determines the citizen's preference for fertility n_t^i relative to consumption c_t^i.

Each adult citizen faces the following resource constraint:

$$c_t^i + \rho^i n_t^i = (1 - \tau^i) y_t^i, \tag{3}$$

in which the parameter $\rho^i > 0$ determines the cost of raising n_t^i children as $\rho^i n_t^i$. These children then become adults in the next period. The tax rate $\tau^i \in (0, 1)$ is levied by political elites who are the ruling class within state i. As Galor (2022, p. 209) writes, "[d]uring the agricultural stage of development, taxes were largely paid in crops." Taking the fiscal policy variables as given, each citizen maximizes utility u_t^i in (2) subject to (1) and (3) to derive the utility-maximizing levels of fertility, consumption and agricultural labor as

$$n_t^i = \frac{\gamma^i (1 - \tau^i) y_t^i}{\rho^i}, \tag{4}$$

$$c_t^i = (1 - \gamma^i)(1 - \tau^i) y_t^i, \tag{5}$$

$$l_t^i = \frac{\alpha}{\beta^i + \alpha} \equiv l^i, \tag{6}$$

where $l^i \equiv \alpha/(\beta^i + \alpha)$ is a composite parameter for agricultural labor and decreasing in leisure preference β^i.

As mentioned, there are N_t^i adult citizens in state i at time t, and each of these citizens has n_t^i children. Therefore, the law of motion for the adult population size in state i is

$$N_{t+1}^i = n_t^i N_t^i = \frac{\gamma^i (1 - \tau^i) y_t^i}{\rho^i} N_t^i, \tag{7}$$

where the second equality uses (4). The (adult) population growth rate in state i is then

$$\frac{\Delta N_t^i}{N_t^i} \equiv \frac{N_{t+1}^i - N_t^i}{N_t^i} = \frac{\gamma^i (1 - \tau^i) y_t^i}{\rho^i} - 1, \tag{8}$$

which is increasing in agricultural output y_t^i and fertility preference γ^i but decreasing in the tax rate τ^i and fertility cost ρ^i.

2.3 Political elites

Each state i is ruled by a small group of political elites. They collect tax revenue $\tau^i y_t^i N_t^i$ from all citizens and allocate a share $\omega^i \in (0, 1)$ of this tax revenue to the provision of productive public goods:

$$G_t^i = \omega^i \tau^i y_t^i N_t^i, \tag{9}$$

which features the usual economies of scale in the provision of public goods within each state. Then, the elites consume the remaining share $1-\omega^i$ of tax revenue as rent-seeking taxation $T_t^i = (1-\omega^i)\tau^i y_t^i N_t^i$.[8]

2.4 Aggregate agricultural production function

Substituting (9) into (1) yields the aggregate production function of agricultural output per capita in state i at time t as

$$y_t^i = \left[(\omega^i \tau^i N_t^i)^\kappa (l^i)^\alpha \left(\frac{Z_t^i}{N_t^i}\right)^{1-\alpha}\right]^{1/(1-\kappa)}, \tag{10}$$

where $l^i \equiv \alpha/(\beta^i + \alpha)$ is the composite parameter for agricultural labor as defined in (6). Agricultural output y_t^i is increasing in the tax rate τ^i and in the share ω^i of tax revenue allocated to public goods G_t^i. Also, for a given amount of land Z_t^i in state i, a larger population size N_t^i has two effects on agricultural output y_t^i per capita. On the one hand, it reduces the amount of land available per citizen $z_t^i = Z_t^i/N_t^i$. On the other hand, it increases the provision of public goods $G_t^i = \omega^i \tau^i y_t^i N_t^i$. Which effect dominates depends on the relative magnitude of the elasticity parameter κ and land intensity $1-\alpha$.

2.5 Land competition

To determine the division of land across states $i \in \{1, ..., m\}$, we use the following conflict success function from the literature on the macrotechnology of conflict; see Hirshleifer (1991, 2000). The amount of land occupied by state i is given by

$$Z_t^i = \frac{\mu^i (N_t^i)^\phi}{\sum_{j=1}^m \mu^j (N_t^j)^\phi} Z, \tag{11}$$

in which $Z > 0$ is a parameter that determines the total amount of land in the economy, $\mu^i > 0$ is a parameter that represents the land-capturing ability of state i, and the parameter $\phi \in [0, 1)$ captures the elasticity of the land ratio between any two states and their relative population size. To see this, we take the ratio of Z_t^i and Z_t^j using (11) and obtain

$$\frac{Z_t^i}{Z_t^j} = \frac{\mu^i}{\mu^j}\left(\frac{N_t^i}{N_t^j}\right)^\phi. \tag{12}$$

If $\phi = 0$, then the land ratio Z_t^i/Z_t^j is simply μ^i/μ^j and the amount of land occupied by state i is determined by exogenous parameters as $Z_t^i = \mu^i Z / \sum_{j=1}^m \mu^j$. In this special case, the population size N_t^i of a state i does not affect the amount of land Z_t^i it captures. In the more general case of $\phi \in (0, 1)$, a state i captures more land Z_t^i as its population size N_t^i increases relative to that of other states N_t^j.

---

[8] Suppose the objective function of the elites at time t is $U_t^i = \lambda^i \ln T_t^i + (1-\lambda^i) \ln N_{t+1}^i$, in which the parameter $\lambda^i \in (0, 1)$ measures their preference for T_t^i. Then, the chosen tax rate τ^i and share ω^i of tax revenue allocated to public goods are $\tau^i = \kappa + (1-\kappa)\lambda^i$ and $\omega^i = \kappa/[\kappa + (1-\kappa)\lambda^i]$, in which an increase in λ^i raises τ^i but decreases ω^i. To isolate the effects of τ^i and ω^i, we treat them as separate policy parameters.

3 Population dynamics and interstate competition

Substituting the land-division rule in (11) into the aggregate agricultural production function in (10), we can rewrite the law of motion for population size in state i and its population growth rate in (7) and (8) as

$$N_{t+1}^i = \Omega^i(1-\tau^i)(\tau^i\omega^i N_t^i)^{\kappa/(1-\kappa)} \left[\frac{\mu^i (N_t^i)^\phi}{\sum_{j=1}^m \mu^j (N_t^j)^\phi} \frac{Z}{N_t^i}\right]^{(1-\alpha)/(1-\kappa)} N_t^i, \tag{13}$$

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i(1-\tau^i)(\tau^i\omega^i N_t^i)^{\kappa/(1-\kappa)} \left[\frac{\mu^i (N_t^i)^\phi}{\sum_{j=1}^m \mu^j (N_t^j)^\phi} \frac{Z}{N_t^i}\right]^{(1-\alpha)/(1-\kappa)} - 1, \tag{14}$$

where we have defined the following composite parameter as the Malthusian potential of state i:

$$\Omega^i \equiv \frac{\gamma^i}{\rho^i}(l^i)^{\alpha/(1-\kappa)}, \tag{15}$$

which is increasing in agricultural labor l^i and fertility preference γ^i but decreasing in fertility cost ρ^i. In the following sections, we first consider the special case $\phi = 0$ and then the more general case $\phi \in (0,1)$.

3.1 Independent population dynamics across states

Given $\phi = 0$ in the land-division rule in (11), the population growth rate of state i in (14) simplifies to

$$\frac{\Delta N_t^i}{N_t^i} = \frac{\Omega^i(1-\tau^i)(\tau^i\omega^i)^{\kappa/(1-\kappa)}}{(N_t^i)^{(1-\alpha-\kappa)/(1-\kappa)}} \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{(1-\alpha)/(1-\kappa)} - 1, \tag{16}$$

which is increasing in the state's Malthusian potential Ω^i, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods but an inverted-U function in its tax rate τ^i. The tax rate that maximizes state i's population growth $\Delta N_t^i/N_t^i$ is $\tau^i = \kappa$. On the one hand, a higher tax rate increases productive public goods G_t^i, which raises fertility. On the other hand, a higher tax rate reduces after-tax agricultural output $(1-\tau^i)y_t^i$, which depresses fertility. Combining these two effects gives rise to an overall inverted-U effect on population growth. Whether population dynamics of each state is stable or not depends on the relative magnitude of the elasticity parameter κ and land intensity $1-\alpha$. In this section, we first consider the case $\kappa < 1-\alpha$ and then the other case $\kappa = 1-\alpha$.[9]

If $\kappa < 1-\alpha$, then the population growth rate $\Delta N_t^i/N_t^i$ in state i is decreasing in its population size N_t^i; see Figure 1. In this case, population dynamics is stable, and the population size of state i converges to the following steady-state level:

$$N^i = \left[(\Omega^i)^{1-\kappa}(1-\tau^i)^{1-\kappa}(\tau^i\omega^i)^\kappa \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha}\right]^{1/(1-\alpha-\kappa)}, \tag{17}$$

---

[9] We also discuss the case $\kappa > 1-\alpha$ in Appendix A.

which is increasing in the state's Malthusian potential Ω^i, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods but an inverted-U function in its tax rate τ^i. The tax rate that maximizes state i's steady-state population size N^i is also $\tau^i = \kappa$. Substituting (17) into (10) yields the steady-state level of agricultural output per capita in state i as $y^i = \rho^i/[\gamma^i(1-\tau^i)]$, which can also be obtained by imposing $\Delta N_t^i = 0$ on (8) and is increasing in the tax rate τ^i due to its negative effect on fertility as shown in (4). Proposition 1 summarizes the results.

Proposition 1 *If $\phi = 0$ and $\kappa < 1 - \alpha$, then the population size of state i converges to the stable steady-state level N^i in (17), which is increasing in the state's Malthusian potential Ω^i, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods but an inverted-U function in its tax rate τ^i. The tax rate that maximizes state i's steady-state population size N^i is $\tau^i = \kappa$, which is increasing in the elasticity κ of agricultural output with respect to productive public goods.*

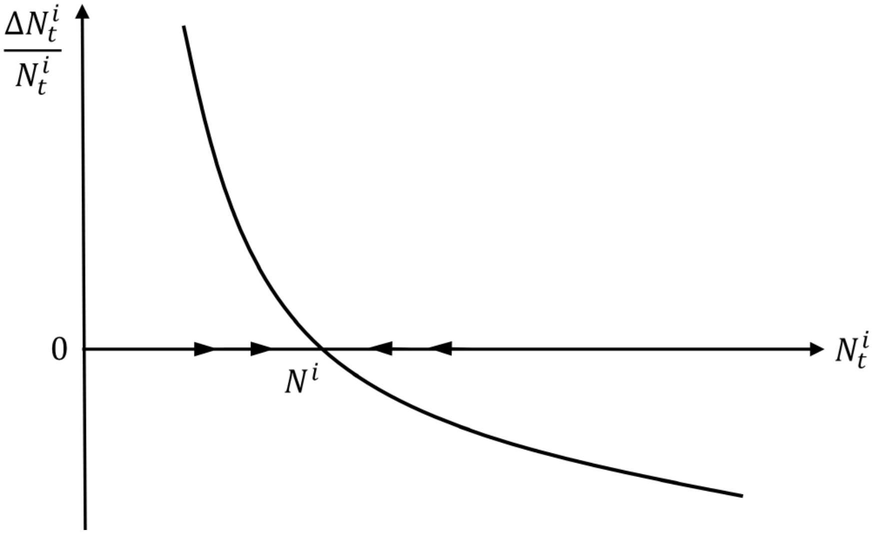

Figure 1: Phase diagram for $\phi = 0$ and $\kappa < 1 - \alpha$

If $\kappa = 1 - \alpha$, then the population growth rate $\Delta N_t^i / N_t^i$ in state i is constant. In this case, the population size N_t^i in state i grows at a constant rate g^i at all time t.

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i(1-\tau^i)(\tau^i\omega^i)^{\kappa/(1-\kappa)} \left(\frac{\mu^i Z}{\sum_{j=1}^{m}\mu^j}\right)^{(1-\alpha)/(1-\kappa)} - 1 \equiv g^i > 0. \tag{18}$$

Although the amount of land available per citizen $z_t^i = Z_t^i/N_t^i$ decreases as population grows, a larger population also increases the provision of public goods $G_t^i = \omega^i\tau^i y_t^i N_t^i$. These two effects offset each other and generate a constant population growth rate g^i, which is increasing in $\{\Omega^i, \mu^i, \omega^i\}$ and an inverted-U function in τ^i as before. Imposing $\kappa = 1 - \alpha$ on (10) yields the steady-state level of agricultural output per capita in state i as $y^i = (\omega^i\tau^i Z^i)^{(1-\alpha)/\alpha} l^i$, where $Z^i = \mu^i Z / \sum_{j=1}^{m} \mu^j$. Interestingly, in this case, y^i is increasing in the tax rate τ^i due to its positive effect on the provision of productive public goods. Furthermore, y^i is also increasing in the share ω^i of tax revenue allocated to public goods, agricultural land Z^i and labor l^i. Proposition 2 summarizes the results.

Proposition 2 *If $\phi = 0$ and $\kappa = 1 - \alpha$, then the population size of state i grows at the constant rate g^i in (18), which is increasing in the state's Malthusian potential Ω^i, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods but an inverted-U function in its tax rate τ^i. The tax rate that maximizes state i's population growth rate g^i is $\tau^i = \kappa$, which is increasing in the elasticity κ of agricultural output with respect to productive public goods.*

3.2 Population dynamics under interstate competition

Given $\phi \in (0, 1)$ in the land-division rule in (11), the law of motion for the population size of state i in (13) can be re-expressed as

$$N_{t+1}^i = \Omega^i (1 - \tau^i)(\tau^i \omega^i)^{\kappa/(1-\kappa)} \left[\frac{\mu^i Z}{\sum_{j=1}^{m} \mu^j (N_t^j)^{\phi}} \right]^{(1-\alpha)/(1-\kappa)} (N_t^i)^{[1-(1-\phi)(1-\alpha)]/(1-\kappa)}, \tag{19}$$

in which we have collected the exponents on N_t^i. In this case, we need the following parameter restriction for the stability of population dynamics: $\kappa \leq (1-\alpha)(1-\phi) \in (0, 1)$. We first consider the case $\kappa < (1-\alpha)(1-\phi)$ and then the other case $\kappa = (1-\alpha)(1-\phi)$.

If $\kappa < (1-\alpha)(1-\phi)$, then all states $i \in \{1, ..., m\}$ coexist in the long run. To see this result, we use (19) to derive the relative population size between any two states i and j as

$$\frac{N_{t+1}^i}{N_{t+1}^j} = \frac{\Omega^i (1 - \tau^i)(\tau^i \omega^i)^{\kappa/(1-\kappa)}}{\Omega^j (1 - \tau^j)(\tau^j \omega^j)^{\kappa/(1-\kappa)}} \left(\frac{\mu^i}{\mu^j} \right)^{(1-\alpha)/(1-\kappa)} \left(\frac{N_t^i}{N_t^j} \right)^{[1-(1-\phi)(1-\alpha)]/(1-\kappa)}, \tag{20}$$

which shows that N_t^i/N_t^j converges to a stable steady state because $\kappa < (1-\alpha)(1-\phi)$ implies that the exponent on N_t^i/N_t^j is less than one; see Figure 2. This steady-state ratio N^i/N^j is given by

$$\frac{N^i}{N^j} = \left[\frac{(\Omega^i)^{1-\kappa} (1 - \tau^i)^{1-\kappa} (\tau^i \omega^i)^{\kappa}}{(\Omega^j)^{1-\kappa} (1 - \tau^j)^{1-\kappa} (\tau^j \omega^j)^{\kappa}} \left(\frac{\mu^i}{\mu^j} \right)^{1-\alpha} \right]^{\frac{1}{(1-\phi)(1-\alpha)-\kappa}}. \tag{21}$$

Given $N_t^i/N_t^j = N^i/N^j$ in (21), we can then rewrite (19) as

$$\frac{\Delta N_t^i}{N_t^i} = \frac{\Omega^i (1 - \tau^i)(\tau^i \omega^i)^{\kappa/(1-\kappa)}}{(N_t^i)^{(1-\alpha-\kappa)/(1-\kappa)}} \left[\frac{\mu^i Z}{\sum_{j=1}^{m} \mu^j (N^j/N^i)^{\phi}} \right]^{(1-\alpha)/(1-\kappa)} - 1, \tag{22}$$

which shows that the population growth rate $\Delta N_t^i/N_t^i$ in state i is decreasing in its population size N_t^i. In this case, population dynamics is stable, and the population size of state i converges to the following steady-state level:

$$N^i = \left\{(\Omega^i)^{1-\kappa}(1-\tau^i)^{1-\kappa}(\tau^i\omega^i)^{\kappa}\left[\frac{\mu^i Z}{\sum_{j=1}^{m}\mu^j(N^j/N^i)^{\phi}}\right]^{1-\alpha}\right\}^{1/(1-\alpha-\kappa)}, \tag{23}$$

in which the relative population size N^i/N^j is given in (21). In summary, N^i is increasing in $\{\Omega^i, \mu^i, \omega^i\}$ and an inverted-U function in τ^i as before but is now decreasing in $\{\Omega^j, \mu^j, \omega^j\}$ and a U-shaped function in τ^j. Substituting (23) and (11) into (10) yields the steady-state level of agricultural output per capita in state i as $y^i = \rho^i/[\gamma^i(1-\tau^i)]$, which can also be obtained from (8) and is increasing in its own tax rate τ^i as before. Proposition 3 summarizes the results.

Proposition 3 *If $\phi \in (0,1)$ and $\kappa < (1-\alpha)(1-\phi)$, then all states $i \in \{1,...,m\}$ coexist in the long run. The population size of each state i converges to its stable steady-state level N^i in (23), which is rising in the state's Malthusian potential Ω^i, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods and an inverted-U function in its tax rate τ^i but falling in the other states' Malthusian potential Ω^j, land-capturing ability μ^j and share ω^j of tax revenue allocated to public goods and a U-shaped function in their tax rates τ^j. The own tax rate that maximizes state i's steady-state population size N^i is $\tau^i = \kappa$, which is increasing in the elasticity κ of agricultural output with respect to productive public goods.*

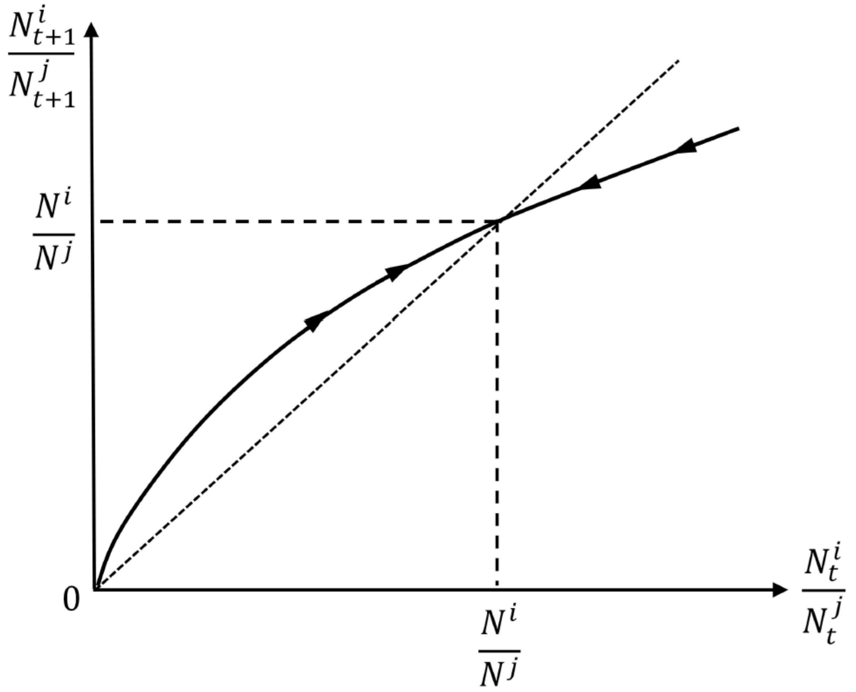

Figure 2: Phase diagram for $\phi \in (0,1)$ and $\kappa < (1-\alpha)(1-\phi)$

If $\kappa = (1-\alpha)(1-\phi)$, then the population growth rate of each state i in (14) can be re-expressed as

$$\frac{\Delta N_t^i}{N_t^i} = \Omega^i(1-\tau^i)(\tau^i\omega^i)^{\kappa/(1-\kappa)}(\mu^i)^{(1-\alpha)/(1-\kappa)}\left[\frac{Z}{\sum_{j=1}^m \mu^j (N_t^j)^\phi}\right]^{(1-\alpha)/(1-\kappa)} - 1. \tag{24}$$

In this case, the state with the largest composite value of $\Omega^i(1-\tau^i)(\tau^i\omega^i)^{\kappa/(1-\kappa)}(\mu^i)^{(1-\alpha)/(1-\kappa)}$ would have the highest population growth rate at all time t. As a result, this state i would be the only surviving state in the long run. It is useful to note that the composite value $\Omega^i(1-\tau^i)(\tau^i\omega^i)^{\kappa/(1-\kappa)}(\mu^i)^{(1-\alpha)/(1-\kappa)}$ is an inverted-U function in the tax rate τ^i, in which the initial positive effect of taxation on interstate competition is due to the importance κ of productive public goods on agricultural productivity. As the population size of this state i converges to a steady state with zero population growth $\Delta N_t^i/N_t^i = 0$, the other states would experience negative population growth $\Delta N_t^j/N_t^j < 0$ for all states $j \neq i$. These states shrink as the land that they occupied become insufficient to sustain nonnegative population growth. Therefore, the population size of all other states would converge to zero, implying the endogenous collapse of these states in the long run, whereas the steady-state population size of the surviving state (i.e., a unified empire) is given by

$$N^i = \left[(\Omega^i)^{1-\kappa}(1-\tau^i)^{1-\kappa}(\tau^i\omega^i)^\kappa Z^{1-\alpha}\right]^{\frac{1}{\phi(1-\alpha)}}, \tag{25}$$

which is increasing in $\{\Omega^i, \omega^i\}$ and an inverted-U function in τ^i as before but becomes independent of μ^i as the empire occupies all the land Z and has no additional land to capture. Substituting (25) and (11) into (10) yields the steady-state level of agricultural output per capita in the surviving state i as $y^i = \rho^i/[\gamma^i(1-\tau^i)]$, which can also be obtained from (8) and is increasing in the tax rate τ^i as before. Proposition 4 summarizes the results.

Proposition 4 *If $\phi \in (0,1)$ and $\kappa = (1-\alpha)(1-\phi)$, then the state i with the largest value of $\Omega^i(1-\tau^i)(\tau^i\omega^i)^{\kappa/(1-\kappa)}(\mu^i)^{(1-\alpha)/(1-\kappa)}$ would be the only surviving state in the long run with a steady-state population size N^i given in (25), which is increasing in the state's Malthusian potential Ω^i and share ω^i of tax revenue allocated to public goods and an inverted-U function in its tax rate τ^i but independent of land-capturing ability μ^i. The population size of all other states $j \neq i$ converges to zero. The tax rate that maximizes the surviving state i's steady-state population size N^i is $\tau^i = \kappa$, which is increasing in the elasticity κ of agricultural output with respect to productive public goods.*

4 Investment in productive public infrastructure

In this section, we consider an extended model with productive public goods as a state variable. In other words, we consider each state i's investment in productive infrastructure. In this case, the law of motion for productive public goods G_t^i is modified from (9) and specified as follows:

$$G_{t+1}^i = \omega^i\tau^i y_t^i N_t^i + (1-\delta)G_t^i, \tag{26}$$

in which $\delta \in (0,1]$ is the depreciation rate of public infrastructure, whereas $\omega^i \tau^i y_t^i N_t^i$ is the tax revenue allocated to public investment in infrastructure. Substituting the agricultural production function in (1) into (26) yields

$$G_{t+1}^i = \omega^i \tau^i (G_t^i)^\kappa (l^i)^\alpha \left(\frac{Z_t^i}{N_t^i}\right)^{1-\alpha} N_t^i + (1-\delta) G_t^i, \tag{27}$$

where $l^i \equiv \alpha/(\beta^i + \alpha)$ from (6) is the composite parameter for agricultural labor in state i.

For tractability, we focus on the special case of $\phi = 0$ in the land-division rule in (11), which simplifies the growth rate of public goods G_t^i from (27) as follows:

$$\frac{\Delta G_t^i}{G_t^i} = \omega^i \tau^i (l^i)^\alpha \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha} \frac{(N_t^i)^\alpha}{(G_t^i)^{1-\kappa}} - \delta, \tag{28}$$

which is increasing in the tax rate τ^i and the share ω^i of tax revenue allocated to public infrastructure investment. For the population growth rate, we substitute the agricultural production function in (1) into (8) to derive

$$\frac{\Delta N_t^i}{N_t^i} = \frac{\gamma^i (1-\tau^i) y_t^i}{\rho^i} - 1 = \overline{\Omega}^i (1-\tau^i) \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha} \frac{(G_t^i)^\kappa}{(N_t^i)^{1-\alpha}} - 1, \tag{29}$$

where $y_t^i = (G_t^i)^\kappa (l^i)^\alpha \left(Z_t^i/N_t^i\right)^{1-\alpha}$ from (1) and $\overline{\Omega}^i \equiv (l^i)^\alpha \gamma^i/\rho^i$ is the composite parameter for Malthusian potential in state i.

From (28), the $\Delta G_t^i = 0$ locus can be expressed as

$$G^i = \left[\frac{\omega^i \tau^i (l^i)^\alpha}{\delta} \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha}\right]^{1/(1-\kappa)} (N^i)^{\alpha/(1-\kappa)}, \tag{30}$$

which describes a positive relationship between G^i and N^i because a larger population size allows for more investment in productive infrastructure due to economies of scale in public goods provision. From (29), the $\Delta N_t^i = 0$ locus can be expressed as

$$G^i = \frac{(N^i)^{(1-\alpha)/\kappa}}{[\overline{\Omega}^i (1-\tau^i)]^{1/\kappa}} \left(\frac{\sum_{j=1}^m \mu^j}{\mu^i Z}\right)^{(1-\alpha)/\kappa}, \tag{31}$$

which also describes a positive relationship between G^i and N^i because a higher level of productive infrastructure leads to a higher level of agricultural productivity and a larger population size. Whether population dynamics of each state is stable or not depends on the relative magnitude of the elasticity parameter κ and land intensity $1-\alpha$. In the rest of this section, we first consider the case $\kappa < 1-\alpha$ and then the other case $\kappa = 1-\alpha$.[10]

---

[10] We also discuss the case $\kappa > 1-\alpha$ in the appendix.

4.1 The extended model with $\kappa < 1 - \alpha$

If $\kappa < 1 - \alpha$, then population dynamics of each state is stable; see Figure 3. In this case, productive public infrastructure G_t^i and population N_t^i converge to their steady-state values respectively, which are jointly determined by (30) and (31) as

$$G^i = \left\{ \left[\frac{\omega^i \tau^i (l^i)^\alpha}{\delta} \right]^{1-\alpha} \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j} \right)^{1-\alpha} [\overline{\Omega}^i (1 - \tau^i)]^\alpha \right\}^{1/(1-\alpha-\kappa)}, \tag{32}$$

$$N^i = \left\{ \left[\frac{\omega^i \tau^i (l^i)^\alpha}{\delta} \right]^{\kappa} \left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j} \right)^{1-\alpha} [\overline{\Omega}^i (1 - \tau^i)]^{1-\kappa} \right\}^{1/(1-\alpha-\kappa)}, \tag{33}$$

which are both increasing in the state's Malthusian potential $\overline{\Omega}^i$, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods and an inverted-U function in its tax rate τ^i as before. Furthermore, the steady-state level of agricultural output per capita is $y^i = \rho^i / [\gamma^i (1 - \tau^i)]$, which can be obtained by imposing $\Delta N_t^i = 0$ on (29) and is increasing in the tax rate τ^i due to its negative effect on fertility.

Suppose $G_0^i < G^i$ and $N_0^i < N^i$ are both located between the $\Delta G_t^i = 0$ locus and the $\Delta N_t^i = 0$ locus. Then, the equilibrium levels of public infrastructure G_t^i and population N_t^i gradually rise towards G^i and N^i in (32) and (33). During this process, holding constant all parameter values, an older state would have a higher level of public infrastructure G_t^i and a larger population N_t^i until it reaches the steady state; see the evidence in Bockstette *et al.* (2002), Chanda and Putterman (2007) and Borcan *et al.* (2018).

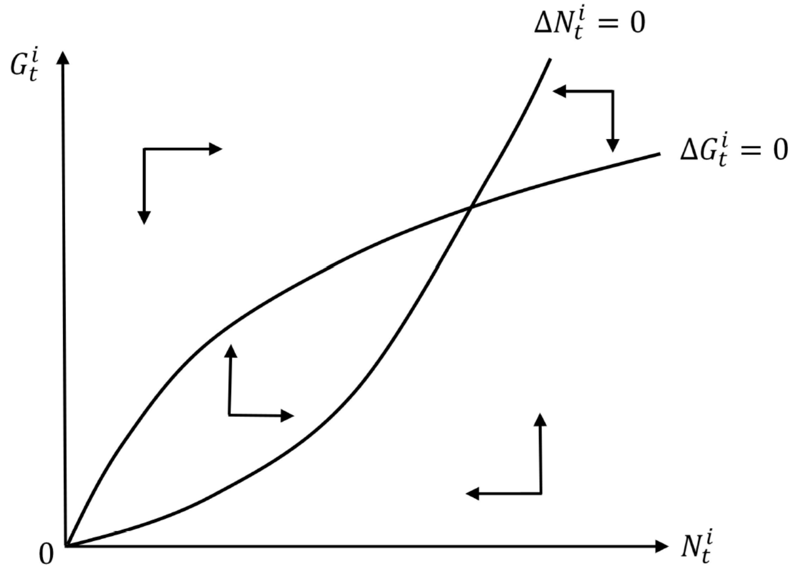

Figure 3: Phase diagram for $\kappa < 1 - \alpha$

Proposition 5 *In the extended model with $\kappa < 1-\alpha$, the level of public infrastructure G_t^i and population size N_t^i of state i converge to the stable steady-state values $\{G^i, N^i\}$ in (32) and (33). The steady-state population size N^i is increasing in the state's Malthusian potential Ω^i, land-capturing ability μ^i and share ω^i of tax revenue allocated to public goods but an inverted-U function in its tax rate τ^i. The tax rate that maximizes state i's steady-state population size N^i is $\tau^i = \kappa$, which is increasing in the elasticity κ of agricultural output with respect to productive public goods.*

4.2 The extended model with $\kappa = 1-\alpha$

If $\kappa = 1-\alpha$, then the dynamics of the ratio G_t^i/N_t^i from (28) and (29) is determined by

$$\frac{\Delta G_t^i}{G_t^i}-\frac{\Delta N_t^i}{N_t^i}=\omega^i\tau^i(l^i)^\alpha\left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha}\left(\frac{N_t^i}{G_t^i}\right)^\alpha-\overline{\Omega}^i(1-\tau^i)\left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha}\left(\frac{G_t^i}{N_t^i}\right)^{1-\alpha}+1-\delta,$$

which is decreasing in G_t^i/N_t^i implying that G_t^i/N_t^i converges to a unique and stable steady state. In this case, productive public infrastructure G_t^i and population N_t^i either grow or shrink towards zero at the same rate in the long run. Equating (28) and (29) yields the steady-state ratio of G_t^i/N_t^i that is implicitly determined by

$$\frac{\Delta G_t^i}{G_t^i}=\frac{\Delta N_t^i}{N_t^i}\Leftrightarrow\overline{\Omega}^i(1-\tau^i)\left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha}\left(\frac{G^i}{N^i}\right)=\omega^i\tau^i(l^i)^\alpha\left(\frac{\mu^i Z}{\sum_{j=1}^m \mu^j}\right)^{1-\alpha}+(1-\delta)\left(\frac{G^i}{N^i}\right)^\alpha, \tag{34}$$

which implies that G^i/N^i is rising in $\{\omega^i, \tau^i\}$ as shown in Figure 4 and simplifies to $G^i/N^i = \omega^i\tau^i(l^i)^\alpha/[\overline{\Omega}^i(1-\tau^i)]$ if $\delta = 1$.

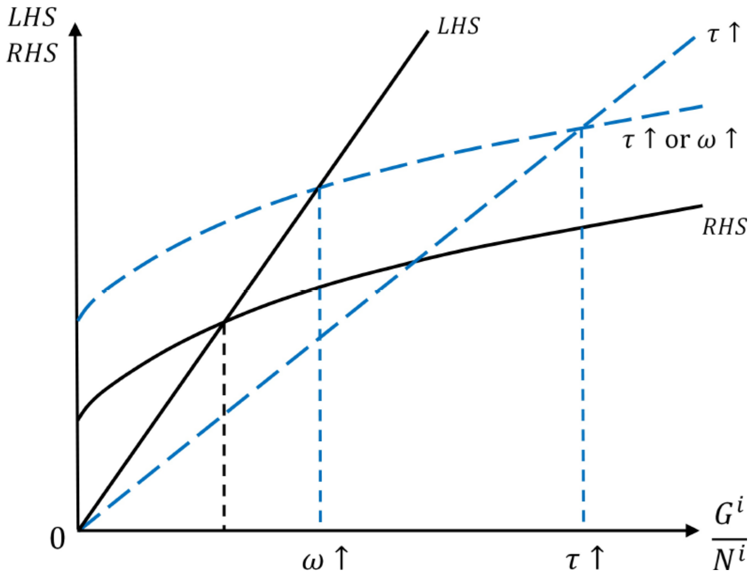

Figure 4: Comparative statics of (34)

Using (1) and $\kappa = 1 - \alpha$, we can express steady-state agricultural output per capita as

$$y^i = \left(\frac{G^i}{N^i}\right)^{1-\alpha} (l^i)^{\alpha} \left(\frac{\mu^i Z}{\sum_{j=1}^{m} \mu^j}\right)^{1-\alpha}, \tag{35}$$

in which G^i/N^i is rising in τ^i. Therefore, y^i is increasing in the tax rate τ^i as before. The steady-state growth rate of public infrastructure G_t^i and population N_t^i is

$$\frac{\Delta G_t^i}{G_t^i} = \frac{\Delta N_t^i}{N_t^i} = \overline{\Omega}^i(1-\tau^i)\left(\frac{\mu^i Z}{\sum_{j=1}^{m} \mu^j}\right)^{1-\alpha} \left(\frac{G^i}{N^i}\right)^{1-\alpha} - 1 \equiv \overline{g}^i, \tag{36}$$

which is initially rising in τ^i (due to its positive effect on G^i/N^i) and subsequently falling in τ^i (due to its negative effect via the term $1 - \tau^i$). G^i/N^i increases as τ^i rises above zero, and $\overline{g}^i$ eventually falls below zero as $\tau^i < 1$ becomes high enough.[11] If $\delta = 1$, then $\overline{g}^i$ has a closed-form solution and is clearly an inverted-U function in τ^i. Figure 5 assumes that $\overline{g}^i$ in (36) is greater than zero, which happens if the following parameter condition holds:

$$\omega^i \tau^i (l^i)^{\alpha} [\overline{\Omega}^i (1-\tau^i)]^{\alpha/(1-\alpha)} \left(\frac{\mu^i Z}{\sum_{j=1}^{m} \mu^j}\right) > \delta. \tag{37}$$

In contrast, Figure 6 assumes that $\overline{g}^i$ in (36) is less than zero, which happens if the inequality in (37) is reversed. Proposition 6 summarizes the results.

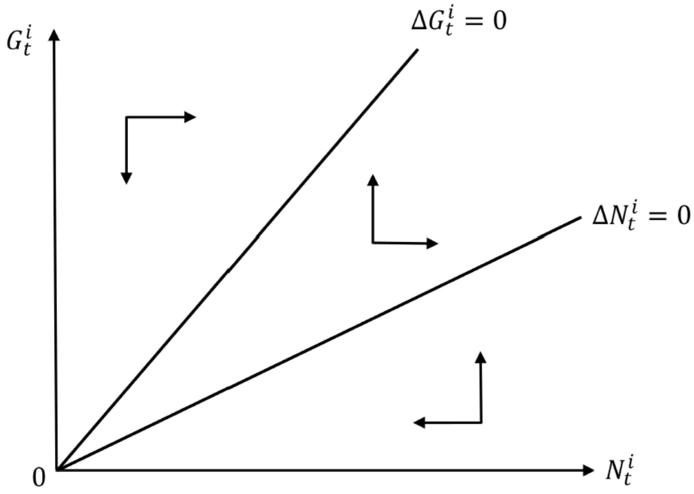

Figure 5: Phase diagram for $\kappa = 1 - \alpha$ and $\overline{g}^i > 0$

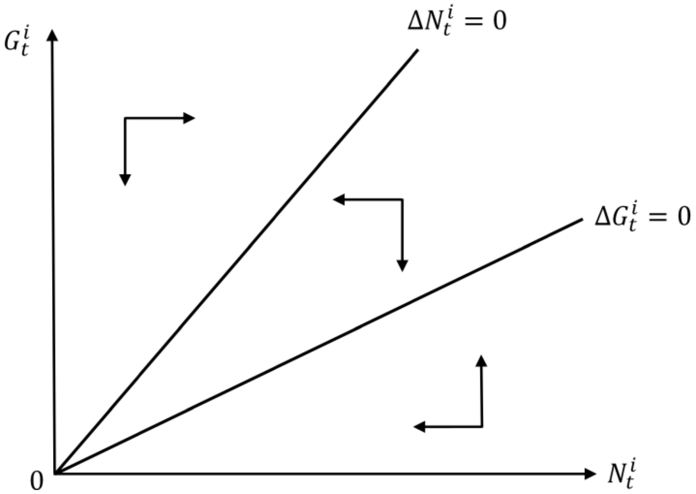

Figure 6: Phase diagram for $\kappa = 1 - \alpha$ and $\overline{g}^i < 0$

---

[11] Appendix B proves that $\overline{g}^i$ is an inverted-U function in τ^i.

Proposition 6 *In the extended model with* $\kappa = 1 - \alpha$*, the level of public infrastructure* G_t^i *and population size* N_t^i *of state* i *grow at the constant rate* $\overline{g}^i$ *in (36), which is an increasing function in the share* ω^i *of tax revenue allocated to public infrastructure investment but an inverted-U function in the tax rate* τ^i*. The tax rate that maximizes state* i*'s population growth rate* $\overline{g}^i$ *is* $\tau^i = \kappa$*, which is increasing in the elasticity* κ *of agricultural output with respect to productive public goods.*[12]

5 Empirical evidence

In all the variants of our Malthusian model, we find that there is a population-maximizing tax rate for each state and that this optimal tax rate is increasing in the state's elasticity of agricultural output with respect to productive public goods. In this section, we use historical data to provide evidence for these theoretical results.

We first use the following specification to examine the inverted-U relationship between taxation and population in the Malthusian era

$$N_j = \varrho_1 \tau_j + \varrho_2 (\tau_j)^2 + \Gamma \Phi_j + \Theta X_j + \zeta_c + \epsilon_j,$$

where N_j is population density in country j in 1500 CE. τ_j denotes historical taxation capacity in country j in 1500 CE. Since tax revenues are not directly observable across countries in the Malthusian era, we use the antiquity index constructed by Bockstette *et al.* (2002) and extended by Borcan *et al.* (2018), as a proxy for historical taxation capacity. This index captures the accumulated administrative infrastructure that underpins fiscal capacity for tax collection.[13] Our theory predicts an inverted-U relationship between taxation and population in the Malthusian era, i.e., $\varrho_1 > 0$ and $\varrho_2 < 0$. In addition, Φ_j denotes historical controls, namely the timing of agriculture transition and the timing of the first settlement by early modern humans. X_j is a set of geographic and agroecological control variables, including absolute latitude, distance to coast or navigable rivers, landlocked status, elevation, land suitability index, arable land share, temperature, and malaria risk. ζ_c denotes continent fixed effects. Finally, ϵ_j is the error term. Table C1 in Appendix C provides the summary statistics.

Table 1 reports the estimation results. Across columns (1)–(4), the coefficient ϱ_1 on taxation is significantly positive, while the coefficient ϱ_2 on its squared term is significantly negative. These results provide evidence of an inverted-U relationship between taxation and population density. Specifically, population initially increases with taxation, but eventually declines as taxation rises. This non-monotonic relationship is also obtained by Borcan *et al.* (2018), and we replicate their results using our data.

Furthermore, our theory predicts that the optimal tax rate is increasing in the elasticity of agricultural output with respect to productive public goods. One determinant of this elasticity is disease ecology. Alsan (2015) shows that the tsetse fly spreads a disease that makes domesticated draft animals sick and often kills them. Because these animals are

---

[12] See Appendix B.

[13] See Borcan *et al.* (2018) for a discussion of how state history reflects the ability to collect taxes.

Table 1: Inverted-U relationship between taxation and population

	Population density in 1500 CE			
	(1)	(2)	(3)	(4)
τ_j	9.559***	6.336***	6.457***	5.394***
	(1.666)	(2.026)	(1.775)	(1.793)
$(\tau_j)^2$	-12.324***	-9.053***	-6.222**	-4.503*
	(3.098)	(3.188)	(2.702)	(2.424)
Historical controls		✓	✓	✓
Geographic & agroecological controls			✓	✓
Continent fixed effects				✓
Observations	154	145	127	127
R^2	0.2542	0.3157	0.7141	0.7545

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses.*

complementary to the plow, a key agricultural technology, plow use becomes less feasible and less productive, which lowers the return to its adoption. This suggests that disease ecology reduces the productivity gains from agricultural technologies and thus affects the elasticity in our model. However, since the tsetse fly is unique to Africa, we use malaria risk in the cross-country regression to obtain a broader sample. Malpede (2023) provides historical evidence that malaria eradication increases agricultural output per farmer even after controlling for agricultural technology adoption and agroecological conditions. This implies that higher malaria risk reduces agricultural productivity conditional on the available agricultural technologies. Motivated by this evidence, we construct a proxy $\kappa_j = 1 - mal_j$ for cross-country variation in the elasticity of agricultural output with respect to productive public goods, where mal_j denotes malaria risk in country j. Specifically, lower malaria risk indicates a lower disease burden and therefore a higher value of κ_j, so that a given increase in productive public investment generates larger gains in agricultural productivity, and vice versa. We use the following specification to test this prediction

$$N_j = \varrho_1 \tau_j + \varrho_2 (\tau_j)^2 + \varrho_3 \tau_j \times \kappa_j + \Gamma \Phi_j + \Theta X_j + \zeta_c + \epsilon_j.$$

The effect of taxation capacity on population density is given by $\partial N_j / \partial \tau_j = \varrho_1 + \varrho_3 \kappa_j + 2\varrho_2 \tau_j$. Therefore, the inverted-U relationship reaches its turning point when $\tau_j = -(\varrho_1 + \varrho_3 \kappa_j)/2\varrho_2$. Our theory predicts that $\varrho_1 > 0$, $\varrho_2 < 0$ and $\varrho_3 > 0$, implying that a higher elasticity of agricultural output with respect to productive public goods (i.e., a larger κ_j) leads to a higher optimal tax rate.

Table 2 reports estimation results for the relationship between the optimal tax rate and the elasticity of agricultural output with respect to productive public goods. Across column (1)-(4), the coefficient ϱ_1 on taxation remains significantly positive, and the coefficient ϱ_2 on its squared term remains significantly negative. More importantly, the coefficient on the interaction term is significantly positive, indicating that the positive effect of taxation is stronger as κ_j increases. This finding suggests that greater returns to productive public investment, as captured by a higher value of κ_j, are associated with a higher optimal tax rate, given by $-(\varrho_1 + \varrho_3 \kappa_j)/2\varrho_2$.

Table 2: Returns to productive public goods and optimal tax rate

	Population density in 1500 CE			
	(1)	(2)	(3)	(4)
τ_j	7.117***	4.743***	4.296***	3.751**
	(1.594)	(1.757)	(1.566)	(1.689)
$(\tau_j)^2$	-15.906***	-12.215***	-8.195***	-5.940**
	(3.141)	(3.206)	(2.436)	(2.352)
$\tau_j \times \kappa_j$	6.236***	4.974***	4.500***	3.585***
	(1.454)	(1.354)	(1.126)	(1.105)
Historical controls		✓	✓	✓
Geographic & agroecological controls			✓	✓
Continent fixed effects				✓
Observations	148	143	127	127
R^2	0.3366	0.3827	0.7432	0.7688

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors are reported in parentheses.*

6 Conclusion

In this study, we have developed a Malthusian growth model with early state formation and interstate competition to explore how the capacity to collect tax revenue and provide productive public goods affects the population growth rate and the survival of early states. Within this growth-theoretic framework, our results can be summarized as follows. We find that population of an agricultural state is initially increasing and subsequently decreasing in its tax rate on agricultural output. Furthermore, we also find that the population-maximizing tax rate is increasing in the elasticity of agricultural output with respect to productive public goods, demonstrating the importance of productive public infrastructure in early state formation. As Galor (2022, p. 209) writes, “[s]tructured polities were able to fund armies, provide public services, impose law and order, invest in human capital and enforce commercial contracts, all of which fostered technological progress and economic growth.” Finally, we have also used historical data to provide empirical evidence for the different implications on population and the optimal tax rate.

References

[1] Ahlerup, P., and Olsson, O., 2012. The roots of ethnic diversity. *Journal of Economic Growth*, 17, 71-102.

[2] Alsan, M., 2015. The effect of the tsetse fly on African development. *American Economic Review*, 105, 382-410.

[3] Ashraf, Q., and Galor, O., 2011. Dynamics and stagnation in the Malthusian epoch. *American Economic Review*, 101, 2003-2041.

[4] Ashraf, Q., and Galor, O., 2013. The "Out of Africa" hypothesis, human genetic diversity, and comparative economic development. *American Economic Review*, 103, 1-46.

[5] Ashraf, Q., and Galor, O., 2018. The macrogenoeconomics of comparative development. *Journal of Economic Literature*, 56, 1119-1155.

[6] Barro, R., 1990. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98, S103-S125.

[7] Barro, R., and Sala-i-Martin, X., 1992. Public finance in models of economic growth. *Review of Economic Studies*, 59, 645-61.

[8] Bockstette, V., Chanda, A., and Putterman, L., 2002. States and markets: The advantage of an early start. *Journal of Economic Growth*, 7, 347-369.

[9] Borcan, O., Olsson, O., and Putterman, L., 2018. State history and economic development: Evidence from six millennia. *Journal of Economic Growth*, 23, 1-40.

[10] Brander, J. A., and Taylor, M. S., 1998. The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. *American Economic Review*, 88, 119-138.

[11] Chanda, A., and Putterman, L., 2007. Early starts, reversals and catch-up in the process of economic development. *Scandinavian Journal of Economics*, 109, 387-413.

[12] Chatterjee, S., and Turnovsky, S.J., 2012. Infrastructure and inequality. *European Economic Review*, 56, 1730-1745.

[13] Chu, A., 2010. Nation states vs. united empire: Effects of political competition on economic growth. *Public Choice*, 145, 181-195.

[14] Chu, A., 2025. *Human Origins and Evolution in a Malthusian Economy*. World Scientific Publishing.

[15] Chu, A., Peretto, P., and Furukawa, Y., 2024. Political fragmentation versus a unified empire in a Malthusian economy. *Journal of Economic Behavior and Organization*, 222, 284-293.

[16] Collins, J., Baer, B., and Weber, E. J., 2014. Economic growth and evolution: Parental preference for quality and quantity of offspring. *Macroeconomic Dynamics*, 18, 1773-1796.

[17] Dalgaard, C.-J., and Strulik, H., 2015. The physiological foundations of the wealth of nations. *Journal of Economic Growth*, 20, 37-73.

[18] de la Croix, D., and Dottori, D., 2008. Easter Island's collapse: A tale of a population race. *Journal of Economic Growth*, 13, 27-55.

[19] Ehrlich, I., and Lui, F., 1997. The problem of population and growth: A review of the literature from Malthus to contemporary models of endogenous population and endogenous growth. *Journal of Economic Dynamics and Control*, 21, 205-242.

[20] Futagami, K., Iwaisako, T., and Ohdoi, R., 2008. Debt policy rule, productive government spending, and multiple growth paths. *Macroeconomic Dynamics*, 12, 445-462.

[21] Futagami, K., and Mino, K., 1995. Public capital and patterns of growth in the presence of threshold externalities. *Journal of Economics*, 61, 123-146.

[22] Futagami, K., Morita, Y., and Shibata, A., 1993. Dynamic analysis of an endogenous growth model with public capital. *Scandinavian Journal of Economics*, 95, 607-625.

[23] Galor, O., 2005. From stagnation to growth: Unified growth theory. *Handbook of Economic Growth*, 1, 171-293.

[24] Galor, O., 2011. *Unified Growth Theory*. Princeton University Press.

[25] Galor, O., 2022. *The Journey of Humanity: The Origins of Wealth and Inequality*. Dutton.

[26] Galor, O., and Michalopoulos, S., 2012. Evolution and the growth process: Natural selection of entrepreneurial traits. *Journal of Economic Theory*, 147, 759-780.

[27] Galor, O., and Moav, O., 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, 117, 1133-1191.

[28] Galor, O., and Ozak, O., 2016. The agricultural origins of time preference. *American Economic Review*, 106, 3064-3103.

[29] Glomm, G., and Ravikumar, B., 1994. Public investment in infrastructure in a simple growth model. *Journal of Economics Dynamics and Control*, 18, 1173-1188.

[30] Glomm, G., and Ravikumar, B., 1997. Productive government expenditures and long-run growth. *Journal of Economic Dynamics and Control*, 21, 183-204

[31] Hirshleifer, J., 1991. The paradox of power. *Economics and Politics*, 3, 177-200.

[32] Hirshleifer, J., 2000. The macrotechnology of conflict. *Journal of Conflict Resolution*, 44, 773-792.

[33] Lagerlof, N.-P., 2007. Long-run trends in human body mass. *Macroeconomic Dynamics*, 11, 367-387.

[34] Lagerlof, N.-P., 2014. Population, technology and fragmentation: The European miracle revisited. *Journal of Development Economics*, 108, 87-105.

[35] Lagerlof, N.-P., 2021. Multiple steady statehood: The roles of productive and extractive capacities. *Journal of Economic Growth*, 26, 113-152.

[36] Le Fur, T., and Wasmer, E., 2025. Fighting for resources: A unified growth model of the Great Divergence. *Journal of Economic Growth*, forthcoming.

[37] Maebayashi, N., Hori, T., and Futagami, K., 2017. Dynamic analysis of reductions in public debt in an endogenous growth model with public capital. *Macroeconomic Dynamics*, 21, 1454-1483.

[38] Malpede, M., 2023. Malaria and economic activity: Evidence from US agriculture. *American Journal of Agricultural Economics*, 105, 1516-1542.

[39] Malthus, T.R., 1798. *An Essay on the Principle of Population.* Oxford World's Classics.

[40] Mayshar, J., Moav, O., and Pascali, L., 2022. The origin of the state: Land productivity or appropriability?. *Journal of Political Economy*, 130, 1091-1144.

[41] Putterman, L., and Trainor, C., 2006. *Agricultural Transition Year Country Data Set.* Brown University.

[42] Svizzero, S., and Tisdell, C., 2014. Inequality and wealth creation in ancient history: Malthus' theory reconsidered. *Economics and Sociology*, 7, 222-239.

[43] Turnovsky, S.J., 1996. Optimal tax, debt, and expenditure policies in a growing economy. *Journal of Public Economics*, 60, 21-44.

[44] Turnovsky, S.J., 2000. Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics*, 45, 185-210.

Appendix A

We first consider the baseline model with $\phi = 0$ and $\kappa > 1 - \alpha$. In this case, the population growth rate $\Delta N_t^i / N_t^i$ in state i is increasing in its population size N_t^i, and population dynamics becomes unstable. Figure A1 shows that if the initial population size N_0^i of state i is above the threshold value in (17), then the population size rises over time and the population growth rate also keeps rising in the long run. The state keeps growing because expansion in productive public goods and population growth reinforce one another, in a virtuous cycle. In contrast, if the initial population size of state i is below the threshold value in (17), then the population size shrinks over time and converges to zero. In this case, the state collapses.

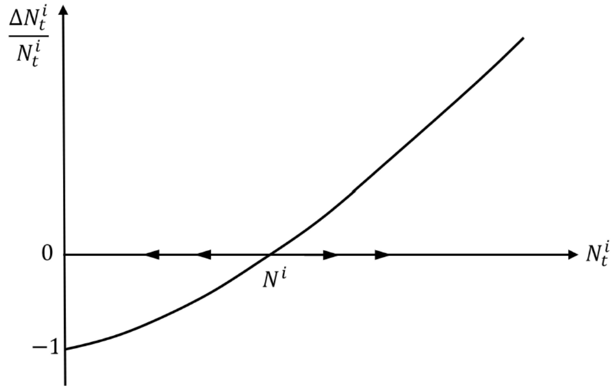

Figure A1: Phase diagram for the baseline model with $\phi = 0$ and $\kappa > 1 - \alpha$

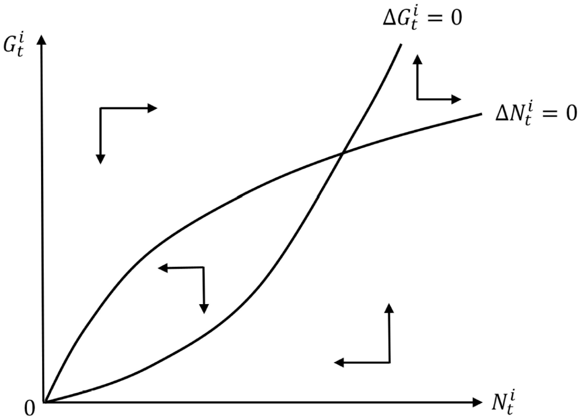

Figure A2: Phase diagram for the extended model with $\phi = 0$ and $\kappa > 1 - \alpha$

We now consider the extended model with $\phi = 0$ and $\kappa > 1 - \alpha$. In this case, population dynamics of each state is unstable. Figure A2 shows that productive public infrastructure G_t^i and population N_t^i may keep rising or shrink towards zero in the long run, depending on the initial levels of public infrastructure G_0^i and population N_0^i at time 0. In other words, holding constant all the parameter values, a state with a lower initial level of public infrastructure G_0^i and a smaller initial population N_0^i is more likely to collapse in the long run (i.e., $\{G_t^i, N_t^i\} \to 0$). In contrast, a state with a higher initial level of public infrastructure G_0^i and a larger initial population N_0^i is more likely to expand indefinitely because public infrastructure expansion and population growth reinforce one another, in a virtuous cycle.

In both models with $\kappa > 1 - \alpha$, a rising (falling) population growth rate $\Delta N_t^i / N_t^i = \gamma^i (1 - \tau^i) y_t^i / \rho^i - 1$ implies a rising (falling) level of agricultural output per capita in the long run, which is inconsistent with evidence of a stationary level of agricultural output per capita in the Malthusian epoch. Therefore, we do not consider $\kappa > 1 - \alpha$ as an empirically relevant parameter space.

Appendix B

This appendix considers the extended model with $\kappa = 1 - \alpha$. To simplify notation, we use $Z^i = \mu^i Z / \sum_{j=1}^{m} \mu^j$. We can then rewrite equations (34) and (35) as

$$\bar{\Omega}^i (1-\tau^i) \left(Z^i\right)^{1-\alpha} \left(\frac{G^i}{N^i}\right) = \omega^i \tau^i (l^i)^\alpha \left(Z^i\right)^{1-\alpha} + (1-\delta) \left(\frac{G^i}{N^i}\right)^{\alpha}; \tag{B1}$$

$$\bar{g}^i = \bar{\Omega}^i (1-\tau^i) \left(Z^i\right)^{1-\alpha} \left(\frac{G^i}{N^i}\right)^{1-\alpha} - 1. \tag{B2}$$

Using (B1), we obtain

$$\frac{\partial \left(G^i/N^i\right)}{\partial \tau^i} = \frac{\omega^i (l^i)^\alpha \left(Z^i\right)^{1-\alpha} + \bar{\Omega}^i \left(Z^i\right)^{1-\alpha} \left(\frac{G^i}{N^i}\right)}{\bar{\Omega}^i (1-\tau^i) \left(Z^i\right)^{1-\alpha} - \alpha \left(1-\delta\right) \left(\frac{G^i}{N^i}\right)^{\alpha-1}}. \tag{B3}$$

Substituting (B1) into (B3) yields

$$\frac{\partial \left(G^i/N^i\right)}{\partial \tau^i} = \frac{\omega^i (l^i)^\alpha \left(Z^i\right)^{1-\alpha} + (1-\delta) \left(\frac{G^i}{N^i}\right)^{\alpha}}{\omega^i \tau^i (l^i)^\alpha \left(Z^i\right)^{1-\alpha} + (1-\alpha)\left(1-\delta\right) \left(\frac{G^i}{N^i}\right)^{\alpha}} \frac{G^i/N^i}{1-\tau^i}. \tag{B4}$$

We can then derive

$$\frac{\partial \bar{g}^i}{\partial \tau^i} = \bar{\Omega}^i \left(Z^i\right)^{1-\alpha} \left[(1-\alpha)(1-\tau^i) \left(\frac{G^i}{N^i}\right)^{-\alpha} \frac{\partial (G^i/N^i)}{\partial \tau^i} - \left(\frac{G^i}{N^i}\right)^{1-\alpha} \right]. \tag{B5}$$

Substituting (B4) into (B5), we obtain

$$\frac{\partial \bar{g}^i}{\partial \tau^i} = \bar{\Omega}^i \left(Z^i\right)^{1-\alpha} \left(\frac{G^i}{N^i}\right)^{1-\alpha} \frac{\omega^i (l^i)^\alpha \left(Z^i\right)^{1-\alpha} \left(1-\alpha-\tau^i\right)}{\omega^i \tau^i (l^i)^\alpha \left(Z^i\right)^{1-\alpha} + (1-\alpha)\left(1-\delta\right) \left(\frac{G^i}{N^i}\right)^{\alpha}}. \tag{B6}$$

This expression implies that $\bar{g}^i$ is an inverted-U function of τ^i, and the population-maximizing tax rate is $\tau^i = 1 - \alpha = \kappa$.

Appendix C

Table C1: Summary statistics

Variables	Observations	Mean	Sd	Min	Max
Log population per square kilometer	154	0.905	1.461	-3.817	3.842
Antiquity index	154	0.141	0.161	0.000	0.760
Log years since first settlement	152	3.574	1.210	-0.357	5.072
Log years since agricultural transition	146	1.223	0.812	-3.270	2.303
Absolute latitude	154	26.740	17.800	0.4221	67.470
Log distance to coast or navigable rivers	147	5.127	1.262	2.073	7.777
Landlocked (dummy)	133	0.226	0.420	0.000	1.000
Log elevation	147	6.080	0.971	2.216	8.066
Land suitability index	144	0.379	0.248	0.000	0.960
Arable land share	153	0.158	0.138	0.001	0.621
Temperature	154	18.070	8.396	-7.929	28.640
Malaria risk	148	0.323	0.428	0.000	1.000

Data source: Ashraf and Galor (2013) for population density. Putterman and Trainor (2006) for the timing of agricultural transition. Ahlerup and Olsson (2012) for the timing of the first settlement by modern humans. Borcan et al. (2018) for other variables.